ADVANCED PLACEMENT CALCULUS AB
SECOND EXAM ——FIRST QUARTER

You *must be experts* on: **1. Definition of Continuity** **2. Definition of the derivative (both)** **3. Intermediate Value Thm** **4. and the *proper use* of the above 3 items** ***before this second exam of the quarter !***			EXAM Ch 1 --> Ch 4.4 #1 R/C Pg 77 - 84 #2 Pg 84 1 - 6, 11, 13, 29 **Review of Trig** 9/26	# 3 Pg 85 - 86 17, 19 - 28 **More trig** 9/27
# 4 Pg 89 1 - 10 **Important lecture** $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ 9/30	# 5 Pg 89 11 - 17, 19, 20 #6 R/C pg 90 - 95 **More of the same** **Have previous HW** **pblms 4 –> 10 put on chalkboards.** 10/1	# 7 Pg 95 - 96 1 - 24, 28, 30 **Def of Cont.** **Intermediate Value Thm** 10/2	# 8 GQ EXP 1 - 5 GQ Trg 1, 2, 3 Pg 98 - 99 GQ Ch 4 1 - 8 Pg 99 RE 2 - 5, 7, 8 **Students to put-up pblms from #7 10 - 21.** **Also discuss pblm 30** 10/3	# 9, #10 Pg 100 Ch 4 REV 11 - 15 Pg 100 - 102 RE Ch 1 - 4 1 - 8, 10,11, 10/4
# 11 Pg 100 - 102 RE Ch 1 - 4 13 - 19, 21 – 27, 31 - 38 10/ 7	begin "Continuous Enjoyment 2nd Edition" **Each** problem requires comments. brief statement describing methods #12 #13 pblms 1 - 27 10/8	#14, #15 "Continuous Enjoyment 2nd Edition" problems 28 - 53 10/ 9	"Continuous Enjoyment 2nd Edition" finish 10/ 10	REVIEW 10/ 11
EXAM **CHAPTERS 1 —> 4** 10/ 14		**2002**		

"Continuous Enjoyment -- 2nd Edition"

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

1) If $\sec x$ is positive then it can be expressed in terms of $\sin x$ as

a) $\frac{1}{\sin x}$ b) $\frac{1}{1-\sin^2 x}$ c) $\frac{1}{\sqrt{1-\sin^2 x}}$ d) $\frac{\sin x}{1-\sin x}$ e) $\frac{1}{1-\sin x}$

2) Which of the following functions is even?

I. $f(x)=\sin(x^2-1)$ II. $g(x)=\cos(x^3-2)$ III. $h(x)=(x-3)^2$

a) I only b) II only c) III only d) II and III only e) I, II, and III

3) Which of the following functions has a vertical asymptote at $x=\frac{\pi}{2}$?

I. $f(x)=\sin\left(x-\frac{\pi}{2}\right)$ II. $g(x)=\tan x$ III. $h(x)=\cot x$

a) I only b) II only c) III only d) II and III only e) I and III only

4) If the angle from the x-axis to a given line is 30°, then the slope of the line is:

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{3}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) 1

5) The period of the function $y=3\tan\left(\frac{2}{3}x-\pi\right)$ is

a) $\frac{2\pi}{3}$ b) π c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$ e) 3π

6) If $f(x)=\frac{\sin x-x}{1-\cos x}$ then $f(-x)=$

a) $\frac{\sin x+x}{1+\cos x}$ b) $\frac{\sin x-x}{1-\cos x}$ c) $\frac{x-\sin x}{1-\cos x}$ d) $\frac{-\sin x-x}{1-\cos x}$ e) $\frac{\sin x-x}{1+\cos x}$

7) Which of the following functions is neither even nor odd?

a) $f(x)=\tan x$ b) $f(x)=\sin^2 x$ c) $f(x)=\cos|x|$ d) $f(x)=\frac{1-\cos x}{\sin x}$ e) $f(x)=\sec x+\csc x$

8) If $f(x)=x^2$ then $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=$

a) a^2 b) $2a$ c) a^2+2a d) a^2-2a e) 0

9) The equation of the tangent line to $f(x)=-\frac{3}{x}$ at the point where $x=3$ is:

a) $y=\frac{1}{3}x-2$ b) $y=-\frac{1}{3}x$ c) $y=\frac{1}{3}$ d) $y=-3x+8$ e) $y=-3x+10$

10) $\lim_{x\to 1}\frac{x-1}{x^2+2x-3}=$

a) 0 b) no limit c) ∞ d) $\frac{1}{2}$ e) $\frac{1}{4}$

11) If $f(x) = 2 - \left|4 + \frac{x}{2}\right|$ then $f(x)$ is not differentiable at $x =$

a) -2 b) 2 c) 0 d) -8 e) 8

12) $\lim_{x \to -\frac{1}{3}} \frac{9x^2 - 1}{3x + 1} =$

a) 0 b) Nonexistent c) -1 d) -2 e) $-\frac{1}{3}$

13) $\lim_{x \to 2} \frac{\sqrt{x+14} - 4}{x - 2}$

a) 0 b) 8 c) $\frac{1}{8}$ d) 4 e) $\frac{1}{4}$

14) Let $f(x) = \frac{x - 7}{\sqrt{x - 3} - 2}$ which of the following statements are true?

I. $\lim_{x \to 7} f(x) = 4$ II. $f(7)$ is undefined. III. $\lim_{x \to 7} f(x) = f(7)$

a) I only b) II only c) III only d) I and III only e) I and II only

15) Given $f(x) = 3x - 5$ and $g(x) = 3x^3 - 5x^2 - 3x + 5$. Find $\lim_{x \to \frac{5}{3}} \frac{g(x)}{f(x)}$.

a) $\frac{34}{9}$ b) $\frac{16}{9}$ c) 0 d) $\frac{5}{3}$ e) $\frac{16}{3}$

16) If $f(x) = \frac{3x - 1}{2x}$ then $\lim_{x \to 0} f(x) =$

a) 0 b) ∞ c) -1 d) $-\frac{1}{2}$ e) DNE

17) Evaluate: $\lim_{x \to 5} \sqrt{\frac{x^2 - 9}{x - 3}}$

a) 8 b) $2\sqrt{2}$ c) 5 d) $\sqrt{2}$ e) 2

18) Given $f(x) = \begin{cases} 3x - 2 & \text{if } x \geq 5 \\ -x^2 + 3 & \text{if } 0 < x < 5 \\ -x^3 + 1 & \text{if } x \leq 0 \end{cases}$. Find $\lim_{x \to 5^-} f(x)$.

a) 13 b) 28 c) -22 d) -124 e) None of These

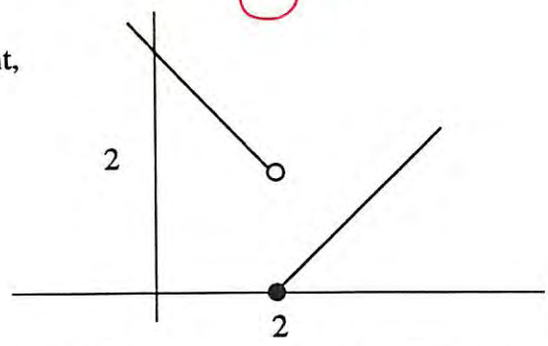
19) Given $f(x) = \frac{|2x - 4|}{x - 2}$. Find $\lim_{x \to 2^+} f(x)$.

a) DNE b) 0 c) 2 d) 1 e) 3

20) Given $f(x)=\begin{cases}\sqrt{(x^2-4)} & : \ x\le -2 \text{ or } x\ge 2\\ x^2-2 & ; \ -2<x<2\end{cases}$. Find $\lim_{x\to -2^-} f(x)$.

a) 2 b) -2 c) 4 d) 0 e) 1

21) From the graph of $f(x)$ shown on the right, which is necessarily true?



I. $\lim_{x\to 2^-} f(x)$ DNE

II. $\lim_{x\to 2^+} f(x)=0$

III. $\lim_{x\to 2} f(x)=0$

a) I only b) II only c) III only d) I and III only e) I and II only

22) Given $f(x)=\dfrac{x^2+2x}{x(x+2)^2(x-1)^4}$, then $\lim_{x\to a} f(x)=\infty$ for $a=$

a) $0,-2,1$ b) $0,1$ c) $1,-2$ d) $-1,2,0$ e) $-1,2$

23) The equation(s) of the vertical asymptote(s) of $f(x)=\dfrac{2x-5}{25-4x^2}$ is (are)

a) $y=\frac{5}{2}$ b) $x=-\frac{5}{2}$ c) $x=\frac{5}{2},\ x=-\frac{5}{2}$ d) $y=\frac{5}{2},\ y=-\frac{5}{2}$ e) $y=-\frac{5}{2}$

24) $\lim_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-25}$

a) $\frac{7}{10}$ b) DNE c) $\frac{2}{5}$ d) 1 e) 0

25) Which of the following functions has a vertical asymptote at $x=2$?

I. $f(x)=\dfrac{1}{x-2}$ II. $g(x)=\dfrac{x-2}{x-2}$ III. $h(x)=\begin{cases}\dfrac{1}{x-2} & ; \ x<2\\ x+5 & ; \ x\ge 2\end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

26) The equation of the horizontal asymptote of $f(x)=\dfrac{3x^2-2x+5}{7x-2x^2-1}$ is:

a) $y=\frac{3}{7}$ b) $x=\frac{3}{7}$ c) $y=-\frac{3}{2}$ d) $y=-5$ e) $x=-\frac{3}{2}$

27) Find $\lim_{x\to -\infty} \dfrac{x^2+4}{x^4-1}$.

a) 0 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) ∞ e) DNE

28) The equation of the horizontal asymptote to $xy - 2y = 3x - 4$ is:

a) $y = 2$ b) $x = 2$ c) $y = 3$ d) $x = 3$ e) none of these

29) $\lim_{x \to -\infty} \left(-5x + \frac{3}{2x^3} \right) =$

a) 0 b) ∞ c) $-\infty$ d) $\frac{3}{2}$ e) -5

30) The graph of the function $f(x) = \frac{x^2 + 2x}{x^2 - 2x}$ has asymptotes at:

a) $x = 0, x = 2, y = 1$ b) $x = 0, x = 2, y = 0$ c) $x = -2, y = 1$ d) $x = 0, x = -2, y = 1$ e) $x = 2, y = 1$

31) For what value(s) of x does $f(x) = \frac{x^2 - 2}{1 - x - x^2}$ cross its horizontal asymptote?

a) 0 b) $\pm\sqrt{2}$ c) -1 d) 1 e) -2

32) If $f(x) = \frac{x + 4}{2x - 1}$ which of the following values of x guarantees that $\left| f(x) - \frac{1}{2} \right| < \frac{1}{10}$?

a) $x < 20$ b) $x > \frac{46}{45}$ c) $x < 23$ d) $x > 23$ e) $x > 20$

33) Find $\lim_{x \to \infty} \frac{\sqrt{x^2 + 4x + 4}}{2x + 3}$

a) ∞ b) 0 c) $\frac{1}{2}$ d) $\frac{1}{3}$ e) 2

34) Given $f(x) = \begin{cases} 2 - 3x & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ 2x^2 - 5 & \text{if } x > 1 \end{cases}$. Which of the following statements are true?

I. $\lim_{x \to 1^-} f(x) = -1$ II. $f(1) = -1$ III. $\lim_{x \to 1} f(x) = f(1)$

a) I only b) II only c) III only d) I and II only e) I, II, III

35) Determine the value of $a + b$ so that the function $f(x) = \begin{cases} -3ax^2 - 1 & ; x > 1 \\ 3b - 2x & ; x \le 1 \end{cases}$ is continuous at $x = 1$.

a) $\frac{1}{3}$ b) 1 c) $-\frac{1}{3}$ d) -1 e) 0

36) Determine the value of k such that $f(x) = \begin{cases} 3kx - 5 & ; x > 2 \\ 4x - 5k & ; x \le 2 \end{cases}$ is continuous.

a) 3 b) $\frac{13}{11}$ c) $\frac{3}{11}$ d) $-\frac{3}{11}$ e) -3

37) Choose b so $f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & ; x<1 \text{ or } x>1 \\ b & ; x=1 \end{cases}$ is continuous.

a) $b \in \Re$ b) $b \in \{\}$ c) 3 d) 1 e) -1

38) Which of the following statements is **not** true?

a) If $f(x)$ is continuous at $x=a$, then $\lim_{x\to a} f(x)$ exists.

b) If $f(x)$ is continuous at $x=a$, then it is defined at $x=a$.

c) If $f(x)$ is defined at $x=a$, then $f(x)$ is continuous at $x=a$. may be true but doesn't have to be

d) If $\lim_{x\to a} f(x) = f(a)$, then $f(x)$ is continuous at $x=a$

e) If $f(x)$ is a polynomial, then it is continuous at $x=a$.

39) Which of the following functions are continuous on $(-\infty, \infty)$?

I. $f(x) = x^2 + 4x - 3$ II. $f(x) = \dfrac{x+1}{x-1}$ III. $f(x) = \begin{cases} x^2+2x & ; x<2 \\ 2x+4 & ; x\geq 2 \end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

40) The function $f(x) = \begin{cases} -3x^2-1 & ; x>-2 \\ -13 & ; x=-2 \\ -x^2-17 & ; x<-2 \end{cases}$. Which of the following is true?

I. $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$ II. $\lim_{x\to -2^-} f(x)$ DNE III. $f(-2) \neq \lim_{x\to -2^-} f(x)$

a) I only b) II only c) III only d) I and III only e) I, II, III

41) $\lim_{x\to 0} \dfrac{2\sin 3x}{5x} =$

a) $\frac{2}{5}$ b) $\frac{6}{5}$ c) 0 d) DNE e) $\frac{3}{5}$

42) $\lim_{x\to \frac{\pi}{4}} \dfrac{2x-\frac{\pi}{2}}{\cos 2x} =$ let $A = x - \frac{\pi}{4}$; $\lim_{A\to 0} \frac{2A}{\cos 2(A+\frac{\pi}{4})} = \lim_{x\to 0} \frac{2A}{\cos(2A+\frac{\pi}{2})} = \lim_{x\to 0} \frac{2A}{\cos 2A \cos\frac{\pi}{2} - \sin 2A \sin\frac{\pi}{2}} = \frac{2A}{0 - \sin 2A} = -1$

a) 0 b) DNE c) $\frac{\pi}{4}$ d) -1 e) 1

43) $\lim_{x\to\infty} \dfrac{\sin x}{2x} =$ $\lim_{x\to\infty} \frac{-2}{2x} \leq \lim_{x\to\infty} \frac{\sin x}{2x} \leq \lim_{x\to 0} \frac{2}{2x}$; $0 \leq \lim_{x\to\infty} \frac{\sin x}{2x} \leq 0$ Note Not $\lim_{x\to\infty} \frac{1}{2}\cdot\frac{\sin x}{x}$ because $x\to\infty$ Not $x\to 0$

a) 0 b) 1 c) ∞ d) $\frac{1}{2}$ e) 2

44) $\lim_{x\to 0} \dfrac{\tan 2x}{x} =$

a) $\frac{1}{2}$ b) 0 c) ∞ d) 1 e) 2

45) $\lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x} =$

a) DNE b) -1 c) 0 d) ∞ e) 1

46) $\lim_{x \to 0} \left(\sin\frac{1}{x}\right)\left(\cos\frac{1}{x}\right) =$

a) 0 b) 1 c) $\frac{1}{2}$ d) -1 e) DNE

47) Which of the following functions is continuous and differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

48) Which of the following functions is differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

49) Find the value of $a + b$ such that f is differentiable at $x = -1$ if $f(x) = \begin{cases} 3x^2 & ; x < -1 \\ ax + b & ; x \geq -1 \end{cases}$.

a) 3 b) -9 c) -6 d) -3 e) 6

50) If $f(x) = \begin{cases} 3x^2 + 4 & ; x < 2 \\ bx^3 & ; x \geq 2 \end{cases}$, find b so that $f(x)$ is continuous.

a) 3 b) $\frac{1}{2}$ c) -1 d) 1 e) 2

51) If $f(x) = \begin{cases} 3x^2 + 4x & ; x \leq 1 \\ 2x^3 + bx + c & ; x > 1 \end{cases}$, find b and c so that $f(x)$ has a derivative at $x = 1$.

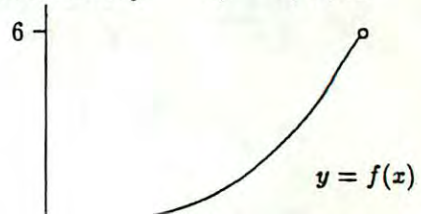
a) b=4, c=0 b) b=1, c=4 c) b=4, c=1 d) b=1, c=0 e) b=1, c=1

52) $f(x) = |x^3|$ then which of the following are true?

I. $f(x)$ is symmetric with respect to the origin.
II. $f(x)$ is continuous for all reals.
III. $f(x)$ is differentiable for all reals.

a) I only b) II only c) III only d) II and III only e) I, II, III

53) The domain of the function $f(x)$ graphed on the right is:



a) $[-2, 5]$ b) $[-3, 6)$ c) $[-2, \infty)$
d) $[-2, 5)$ e) $[-2, 6)$

54. A few graphing problems. Using MODE, select DEGREE and PAR. All other options should be set to left hand option. Using Y=, make $X_{1T} = 8 \sin t$ and $Y_{1T} = 8 \cos t$. Make the viewing window Tmin = 0, Tmax = 360, Tstep = 10, [–15, 15] for the x values and [–10, 10] for the y values.

a) Press GRAPH. Describe the picture you got.
b) Set Tstep = 1. Press GRAPH. Describe the picture you got.
c) Set Tstep = 20. Press GRAPH. Describe the picture you got.
d) Set Tstep = 40. Predict the picture you will get. Press GRAPH. Describe the picture you got.
e) Set Tstep = 60. Predict the picture you will get. Press GRAPH. Describe the picture you got.
f) Set Tstep = 72. Predict the picture you will get. Press GRAPH. Describe the picture you got.
g) Set Tstep = 120. Predict the picture you will get. Press GRAPH. Describe the picture you got.
h) Reset Tmax to 720. Predict the picture you will get. Set Tstep = 144. Press GRAPH. Describe the picture you got.
i) Reset Tmax to 30000. Predict the picture you will get. Set Tstep = 97. Press GRAPH. Describe the picture you got.
j) Summarize and state a conclusion.

55. A few graphing problems. Using MODE, select DEGREE and PAR. All other options should be set to left hand option. Using Y=, make $X_{1T} = 0.003t \sin t$ and $Y_{1T} = 0.003t \cos t$. Make the viewing window Tmin = 0, Tmax = 3000, Tstep = 6, [–15, 15] for the x values and [–10, 10] for the y values.

a) Press GRAPH. Describe the picture you got.
b) As in problem 54 above, select several more Tsteps which will yield interesting pictures. (Hint: You may have to change Tmax.)
c) Summarize and state a conclusion.

56. Write a calculator program which asks the user for two numbers and responds with the average of the two numbers.

57. Write a calculator program which asks the user for two numbers and responds with the square root of the product of the two numbers.

The following problems are intended only as "extras for experts."

*58. Write a calculator program which asks for the slope and y-intercept of a line and responds with the graph of the line.

*59. Write a calculator program which asks for the coordinates of two non-vertical points and responds with the graph of the line.

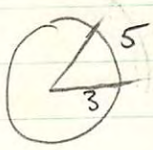
*60. Write a program which asks the user for the number of sides or points and the calculator will produce th

4.5

Pg 84

① a) $90° \Rightarrow \frac{\pi}{2}$ b) $30° \Rightarrow \frac{\pi}{6}$ c) $120° \Rightarrow \frac{2\pi}{3}$ d) $270° \Rightarrow \frac{3\pi}{2}$
e) $360° \Rightarrow 2\pi$

② a) $\frac{3\pi}{4} \Rightarrow 135°$ b) $\frac{\pi}{3} \Rightarrow 60°$ c) $\frac{2\pi}{3} \Rightarrow 120°$ d) $4\pi \Rightarrow 720°$

③ 

$\frac{s}{r} \Rightarrow \frac{s}{R} = \frac{5}{3}$ radians

b) $1° = \frac{\pi}{180}$ $180° \Rightarrow \pi$

$\frac{5}{3}(180)° \Rightarrow \frac{5\pi}{3}$

$95.5 \approx \frac{300°}{\pi} \Rightarrow \frac{5}{3}$ radians.

④

$\theta = \frac{1}{2}$ radian $\therefore$ length of arc on 3 unit circle is $3\left(\frac{1}{2}\right) = \frac{3}{2}$.

⑤ $\pi = 180°$

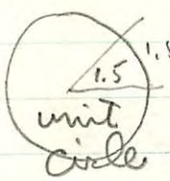
a) $1\ \text{Rad} = \frac{180}{\pi}$

$\therefore 3\ \text{rad} = \frac{540}{\pi} \approx 171.9°$

b) $1°$ into rad. $\approx .017$ rad.

$\pi = 180°$

rad $\frac{\pi}{180} = 1°$

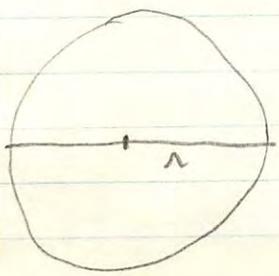
⑥ 

a) $r = 3$ b) $r = 4$ c) $r = 5$

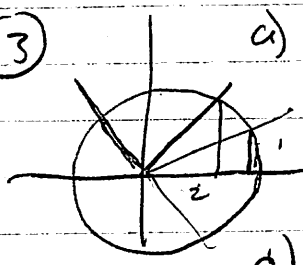
$3(1.5) = 4.5$ $4(1.5) = 6$ $5(1.5) = 7.5$

⑦ a) using protractor measure $\frac{360}{\pi}$ (ie $2 \cdot \frac{180}{\pi}$) degrees
ie $114.6°$

b) using string $2R = \text{diam}$ $C_{\text{circumf}} = 2\pi R$



(8) $\cos\frac{\pi}{6}$ $\sin\frac{\pi}{6}$

(13) 

a) $m=1$ $\theta=45^\circ$ $\frac{\pi}{4}$ rad. $\tan\theta=1 \Rightarrow \theta=\frac{\pi}{4}$

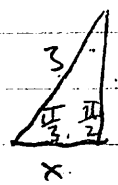
b) $m=\frac{1}{2}$ $\tan\theta=\frac{1}{2}$ $\theta\approx .46$ rad.

c) $m=-1$ $\tan\theta=-1$ $\theta=\frac{3\pi}{4}$

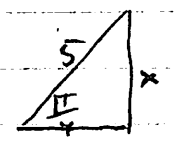
d) $m=2$ $\tan\theta=2$ $\theta\approx 1.107$

e) $m=\sqrt{3}$ $\tan\theta=\sqrt{3}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\theta=60^\circ=\frac{\pi}{3}$ rad.

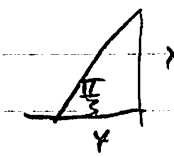
(14) $\frac{OA}{OP}=\cos\theta$ & by Similar Δ $\frac{OA}{OP}=\frac{OC}{OD}=\cos\theta$.
all the rest follow in the same manner.

(17) a) 

$x=\frac{3}{2}$ $\cos\frac{\pi}{3}=\frac{x}{3}$ $\frac{1}{2}=\frac{x}{3}$ $\frac{3}{2}=x$

b) 

$\sin\frac{\pi}{4}=\frac{x}{5}$ $\frac{\sqrt{2}}{2}=\frac{x}{5}$ $x=\frac{5\sqrt{2}}{2}$

c) 

$\tan\frac{\pi}{3}=\frac{x}{4}$ $\sqrt{3}=\frac{x}{4}$ $4\sqrt{3}=x$

(19) a) $(7,3)$ $(4,7)$

$\sqrt{(7-4)^2+(3-7)^2}$

$\sqrt{9+16}$

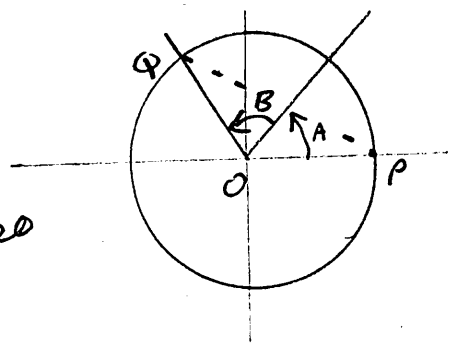
5

b) $\sqrt{x^2+y^2}$

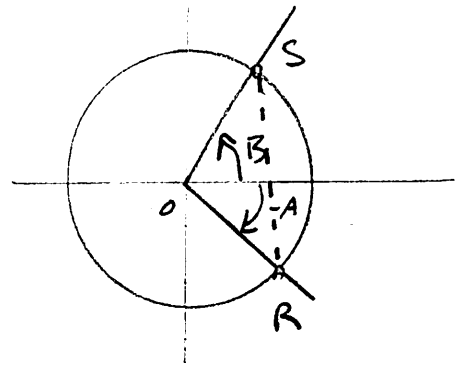
c) $\sqrt{(\cos\theta-1)^2+(\sin\theta-0)^2}$

$\cos^2-2\cos\theta+1+\sin^2\theta$

$\sqrt{2-2\cos\theta}$



20



a) WTS $PQ = RS$

① arc PQ has chord PQ ; arc RS has chord RS.

② arc PQ has $\measuredangle (A+B)$; arc RS has $\measuredangle (A+B)$

The $-A$ measure indicates direction of measure.

The degree measure is $|-A| = A$

③ since circles are $\cong$ & have = arcs then chords are =

b) WTS $Q = (\cos(A+B), \sin(A+B))$. Q has coord $(\cos\theta, \sin\theta)$ since $\theta = A+B$ $Q = (\cos(A+B), \sin(A+B))$

others are similar (easier)

d) WTS $\overline{RS}^2 = 2 - 2\cos A\cos B + 2\sin A\sin B$

$$\overline{RS} = \sqrt{(\cos B - \cos A)^2 + (\sin B + \sin A)^2}$$

$$\overline{RS}^2 = \cos^2 B + \cos^2 A - 2\cos B\cos A + \sin^2 B + \sin^2 A + 2\sin A\sin B$$

$$= 2 - 2\cos B\cos A + 2\sin A\sin B$$

c WTS $\overline{PQ}^2 = 2 - 2\cos(A+B)$

$$\overline{PQ} = \sqrt{(\cos(A+B) - 1)^2 + (\sin(A+B))^2}$$

$$\overline{PQ}^2 = \cos^2(A+B) - 2\cos(A+B) + 1 + \sin^2(A+B)$$

$$= 2 - 2\cos(A+B)$$

E) since $\overline{PQ} = \overline{RS}$, $\overline{PQ}^2 = \overline{RS}^2 \Rightarrow$

$2 - 2\cos(A+B) = 2 - 2\cos B\cos A + 2\sin A\sin B$

$\cos(A+B) = \cos B\cos A - \sin A\sin B$,

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24 Sub $-B$ for B in 20E

↓

25 a) $\cos 2\theta = \cos(\theta+\theta)$ etc.

b) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

c) $\cos\theta = \pm\sqrt{\frac{1+\cos 2\theta}{2}}$

d)

26 WTP $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

$$= \frac{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}}{1 + \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B + \sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\sin(A-B)}{\cos(A-B)}$$

$$= \tan(A-B)$$

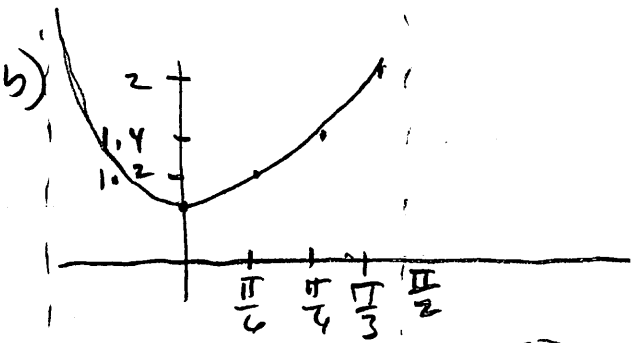
27 a) $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

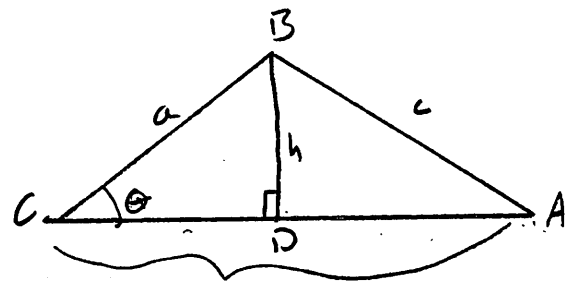
$\sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

$\sec 0 = 1$

b)



28



a) WTS $\overline{CD} = a\cos\theta$

$\frac{CD}{a} = \cos\theta$ Def

$\therefore CD = a\cos\theta$.

WTS $\overline{AD} = b - a\cos\theta$.

$b = \overline{AD} + \overline{CD}$

$b = \overline{AD} + a\cos\theta$

$b - a\cos\theta = \overline{AD}$

b) WTS $a^2 - a^2\cos^2\theta = h^2 = c^2 - (b - a\cos\theta)^2$

$a^2 - \overline{CD}^2 = h^2 = c^2 - \overline{AD}^2$

$a^2 - a^2\cos^2\theta - h^2 = c^2 - (b - a\cos\theta)^2$

c) $a^2 - a^2\cos^2\theta = c^2 - b^2 + 2ab\cos\theta - a^2\cos^2\theta$.

$c^2 = a^2 + b^2 - 2ab\cos\theta$.

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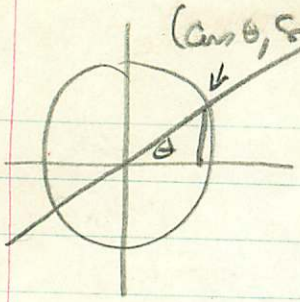
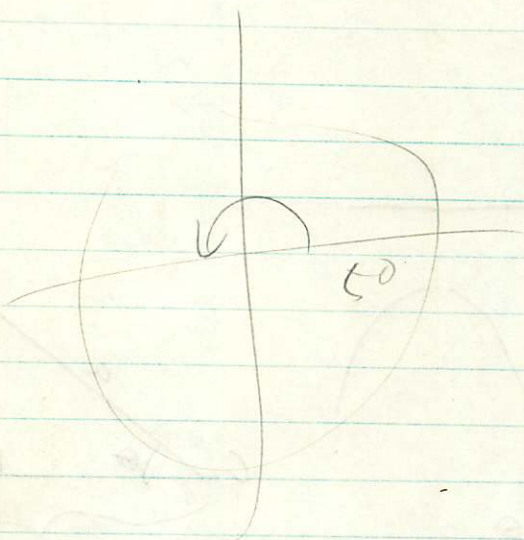
#29.



$$m = \frac{\sin\theta}{\cos\theta} = \frac{\text{rise}}{\text{run}} = \tan\theta = m$$



Lecture #1

Area of shaded / area of circle

area whole circle $= \pi r^2$

Let A = area of shaded part.

θ = any arc

$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$ give arc of that circle / arc length of shaded / whole arc length

$A = \frac{\theta \pi r^2}{2\pi}$

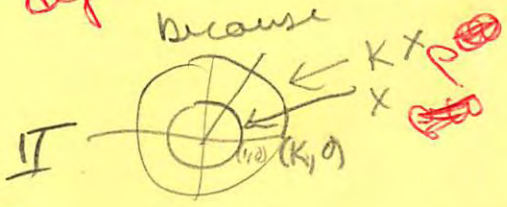
$A = \frac{\theta r^2}{2}$ if we are using a unit circle

$r = 1$

so area of Sector is $\frac{\theta}{2}$.

NOTE
Here The text is using θ to represent a Radian measure instead of the usual degree measure.

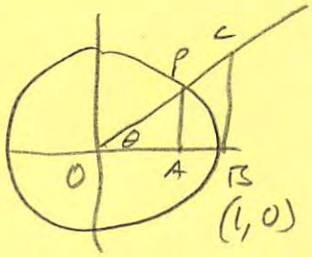
Because



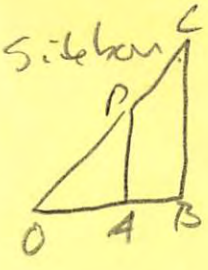
II

WTP $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ $\quad 0 \le \theta < \frac{\pi}{2}$

$P = (\cos\theta, \sin\theta)$



Area $\triangle OAP$ < Area sector OBP < Area $\triangle OBC$



$\frac{1}{2}\sin\theta\cos\theta \le \frac{\theta}{2} \le \frac{1}{2}(\overline{OB})(BC)$

$\frac{1}{2}\sin\theta\cos\theta \le \frac{\theta}{2} \le \frac{1}{2}(1)\left(\frac{\sin\theta}{\cos\theta}\right)$

$\sin\theta\cos\theta \le \theta \le \frac{\sin\theta}{\cos\theta}$

$\cos\theta \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}$

$\triangle OAP \sim OBC$

$\frac{AP}{OA} = \frac{BC}{OB}$

$\frac{\sin\theta}{\cos\theta} = \frac{BC}{1}$

$\lim_{\theta \to 0}$ ( ) =

$1 \le \lim_{\theta \to 0} \frac{\theta}{\sin\theta} \le 1$ ↓

So $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$. Why?

what about $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = ?$

1 as well

So vertical length ($\sin\theta$) is almost the same as arc length (θ) when θ is small. Seems reasonable.

NOTE for the following $\sin\theta$ means θ is measured in degrees.

Notice CAPS

III $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = ?$

$\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta}$

$\lim_{\theta \to 0} \frac{1-\cos^2\theta}{\theta}$

$\lim_{\theta \to 0} \frac{\sin^2\theta}{\theta}$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \cdot \sin\theta$

$1 \cdot 0$

$0.$

~~Finally~~ Conclusion

$\lim_{\star \to 0} \frac{\sin\star}{\star} = 1$

$\lim_{\star \to 0} \frac{1-\cos\star}{\star} = 0$

So ... when θ is small $1-\cos\theta$ is even smaller it $\cos$ is closer to 1 than / θ is to zero.

$$\cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta} \qquad \frac{\cos^2\theta}{\cos^2 x} \quad \frac{\sin\theta}{\theta}$$

$$\frac{1}{\cos\theta} < \frac{\sin\theta}{\theta} < \cos\theta.$$

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θ	90	45	30	10	5	1
$\frac{\sin\theta}{\theta}$	.01	.016	.017	.017	.017	.017

(15) Given $\sin\theta = \sin\frac{\pi\theta}{180}$

WTS $\frac{\sin\theta}{\theta} = \frac{\sin\left(\frac{\pi\theta}{180}\right)}{\frac{\pi\theta}{180}} \cdot \frac{\pi}{180}$

NOTE $\sin\theta$ means θ is in degrees.

a) Obvious

b) $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \frac{\pi}{180}$ as $\theta \to 0$ $\frac{\pi\theta}{180} \to 0$ $\therefore$

$$\lim_{\theta\to 0} \frac{\sin\frac{\pi\theta}{180}}{\frac{\pi\theta}{180}} = 1$$

$$\lim_{\theta\to 0} \frac{\sin\frac{\pi\theta}{180}}{\frac{\pi\theta}{180}} \cdot \frac{\pi}{180} = 1 \cdot \frac{\pi}{180} = \frac{\pi}{180}$$

16. θ is measure in degrees.

a) WTS $\sin\theta = \sin\frac{\theta\pi}{180}$ if

$$\frac{\theta\pi}{180} \approx .017\,\theta.$$

$$\theta(.0175) \approx .017\,\theta.$$

b)

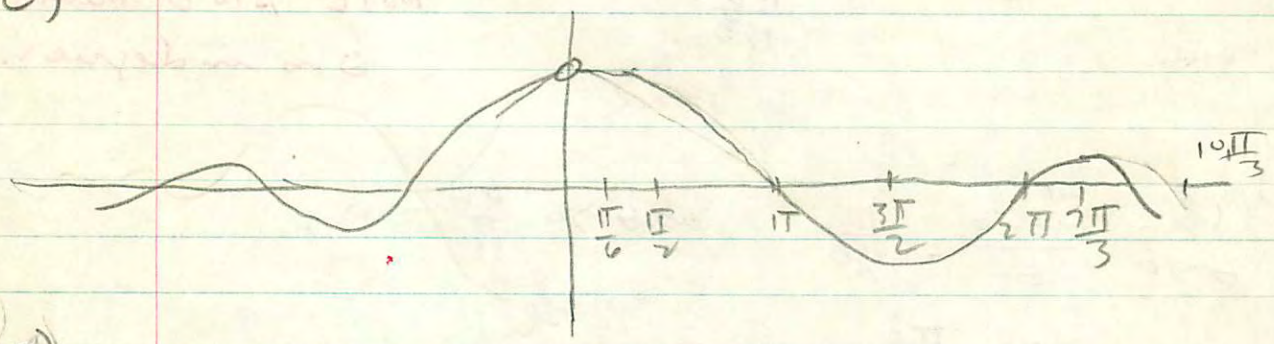
$\theta = 30^\circ$	$\sin\theta$	$.017\theta$
30°	.5	.51
5°	.0872	.085
1°	.017	.017
10°	.1736	.17

(17) $f(x) = \frac{\sin x}{x}$ a) D_f $x \in \mathbb{R}$ $x \neq 0$

b)

x	$\frac{\pi}{6}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$f(x)$	.955	.637	0	-.212	0

c)



(19) a) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ b) $\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$

$$\lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h} =$$

$$\lim_{h \to 0} \frac{-\sin x (1 - \cos h)}{h} + \frac{\sin h \cos x}{h} =$$

$\frac{-\sin x (1-\cos h)}{h} \Rightarrow 0$

$+ \frac{\sin h}{h} \cos x \Rightarrow 1$

$= \cos x$

(20) $\lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h} = \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$

$$\lim_{h \to 0} -\frac{\cos x (1 - \cos h}{h} - \frac{\sin h}{h} \sin x$$

$\frac{\cos x (1-\cos h}{h} \Rightarrow 0$

$\frac{\sin h}{h} \Rightarrow 1$, $\sin x = -\sin x$,

4.6
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① $r=3$ $\theta=\frac{\pi}{2}$ area of sector $=\frac{\theta r^2}{2}$

a) $\frac{9\cdot\pi}{4}$ b) $\frac{\theta}{2}$ c) $\frac{4\theta}{2}=2\theta$

② a) $\frac{\frac{3}{2}\cdot\frac{\pi}{4}}{2}=\frac{9\pi}{2}$ b) $\frac{3\cdot 36}{2}=54$ c) $\frac{\pi}{4}\cdot 36=\frac{9\pi}{2}$

③

θ	$\frac{\pi}{2}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$
$\frac{\sin\theta}{\theta}$	.64	.90	.95

$\theta\to 0$

④ $\lim_{\theta\to 0}\frac{\sin 3\theta}{3\theta}=1$

⑤ $\lim_{\theta\to 0}\frac{1-\cos\theta}{\sin\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}\Rightarrow$

2nd method

$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}\cdot\frac{\theta}{\sin\theta}=0\cdot 1=0$

$\lim_{\theta\to 0}\frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}=$

$\lim_{\theta\to 0}\frac{\sin\theta}{1+\cos\theta}=\frac{0}{2}=0$

⑥ $\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{1}{\cos x}=1\cdot 1=1$

⑦ $\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{3x}{\sin 3x}\cdot\frac{2}{3}$ $\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}\overset{\text{QR}}{=}\lim_{x\to 0}\frac{\sin 2x}{\sin(2x+x)}=\lim_{x\to 0}\frac{2\sin x\cos x}{\sin 2x\cos x+\cos 2x\sin x}$

$1\cdot 1\cdot\frac{2}{3}$

$\lim_{x\to 0}\frac{2\sin x\cos x}{2\sin x\cos^2 x+\sin x(\cos^2 x-\sin^2 x)}=\frac{2}{2+1}=\frac{2}{3}$

⑧ $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}=\lim_{\theta\to 0}\frac{\sin^2\theta}{\theta^2(1+\cos\theta)}=\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{\theta}\cdot\frac{1}{1+\cos\theta}$

$1\cdot 1\cdot\frac{1}{2}=\frac{1}{2}$

⑨ $\lim_{\theta\to 0^+}\frac{1-\cos\theta}{\theta^3}=\lim_{\theta\to 0^+}\frac{1-\cos\theta}{\theta^2}\cdot\frac{1}{\theta}$ limit does not exist

couldn't eliminate last θ

(10) $\lim_{\theta \to \frac{\pi}{2}} \frac{\cos\theta}{\frac{\pi}{2} - \theta}$ let $x = \frac{\pi}{2} - \theta$

$\theta = \frac{\pi}{2} - x$

$\lim_{x \to 0} \frac{\cos\left(\frac{\pi}{2} - x\right)}{x}$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\sin 2\theta + \theta$

$\sin 2\theta \cos\theta + \sin\theta \cos 2\theta$

$2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta)$

(11) $\lim_{\theta \to 0} \frac{\sin 3\theta}{1 - \cos\theta} \cdot \frac{1 + \cos\theta}{1 + \cos\theta}$

$\lim_{\theta \to 0} \frac{\sin 3\theta (1 + \cos\theta)}{1 - \cos^2\theta} = \lim_{\theta \to 0} \frac{\sin 3\theta (1 - \cos\theta)}{\sin^2\theta}$

$\lim_{\theta \to 0} \frac{2\sin\theta\cos^2\theta + \sin\theta(\cos^2\theta - \sin^2\theta)}{1 - \cos\theta}$

$\lim_{\theta \to 0} \frac{\sin\theta(2\cos^2\theta + \cos^2\theta - \sin^2\theta)}{1 - \cos\theta}$

$\lim_{\theta \to 0} \frac{\sin\theta(3\cos^2\theta - 1 + \cos^2\theta)}{1 - \cos\theta}$

$\lim_{\theta \to 0} \frac{-\sin\theta(1 - 4\cos^2\theta)}{1 - \cos\theta} = \lim_{\theta \to 0} -\sin\theta(1 - 2\cos^2\theta)(1 + 2\cos^2\theta)$

$\lim_{\theta \to 0} \frac{3\theta \sin 3\theta}{3\theta(1 - \cos\theta)} = \lim_{\theta \to 0} \frac{3\sin 3\theta}{3\theta} \cdot \frac{1}{\frac{1 - \cos\theta}{\theta}} \Rightarrow 0$

$\Downarrow$

$3 \cdot \frac{1}{0} \Rightarrow \infty$

12. $\lim_{x \to \infty} x \sin\frac{1}{x}$

$\lim_{x \to \infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$ let $\frac{1}{x} = \theta$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

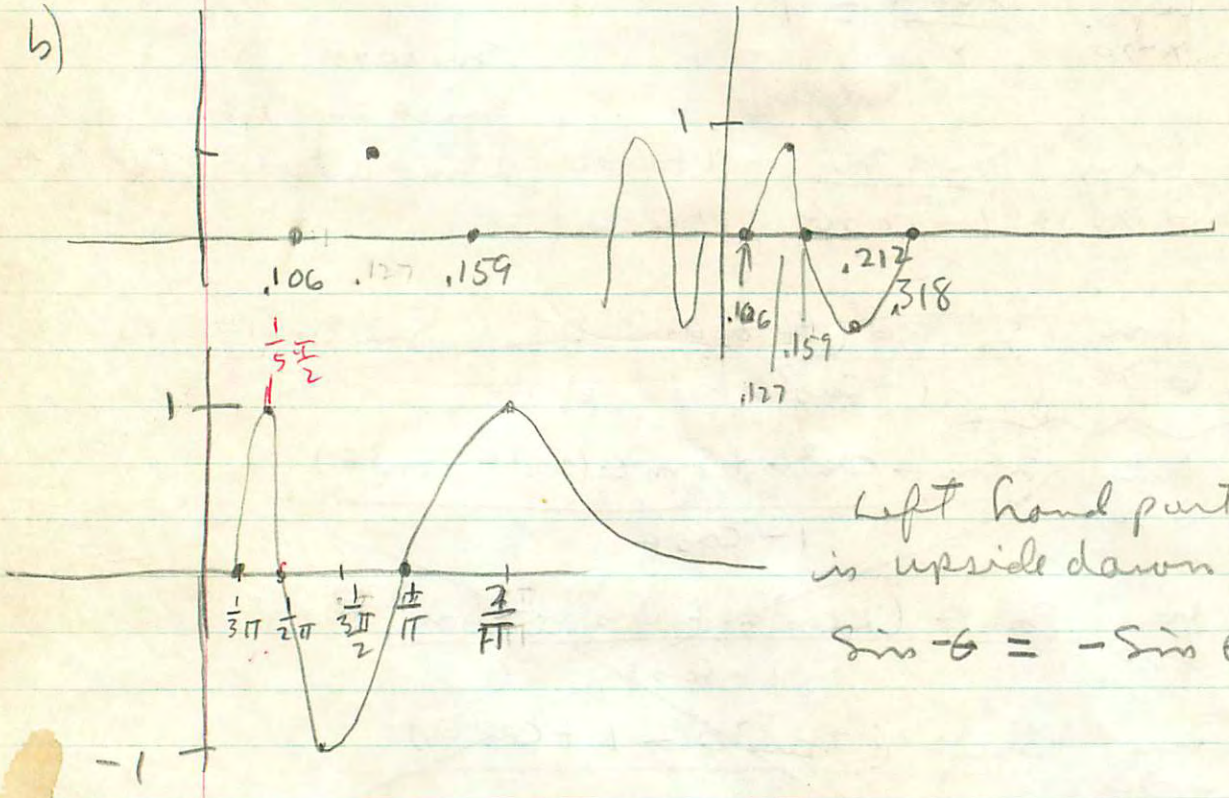
Pg 89

13. $f(x) = \sin \frac{1}{x}$

a)

x	$\frac{1}{3\pi}$	$\frac{1}{5\pi/2}$	$\frac{1}{2\pi}$	$\frac{1}{3\pi/2}$	$\frac{1}{\pi}$	100
$f(x)$	.10	.11	.10	.11	.10	.01

b)



Left hand portion is upside down since $\sin -\theta = -\sin \theta$.

c) limit cannot exist since the curve is oscillating near zero.

d) $\lim_{x \to \infty} \sin \frac{1}{x} = \sin 0 = 0.$

Cal 2nd Exam

Definition of Continuity and Intermediate Value Theorem (*IVT*)

J Mumaugh

Ok.... this is old news... but still --- just to be sure we all are using the same definitions....

Def of Continuity: $\longrightarrow$ is a 3 step process!

$$\lim_{x\to a} f(x) = L \quad = \quad f(a)$$

(2) (3) (1)

(1)

$f(a)$ exists

$f(a)$ must exist or the function is not continuous at $x = a$.

(2)

$$\lim_{x\to a} f(x) = L$$

Note: This implies that

$$\lim_{x\to a^-} f(x) = \ell_1$$

and

$$\lim_{x\to a^+} f(x) = \ell_2$$

and

$$\ell_1 = \ell_2 = L$$

(3)

$L = f(a)$

If (3) is true, then f is continuous at $x = a$.

Question: When do you *have* to approach from each side?
Answer: When the function changes in its nature or description at that point.

Example:

Given $f(x) = |x-2|$. Is $f(x)$ Continuous at $x = 5$? Sure it is. AND at x = 5 nothing about $f(x)$ changes so you would *not* need to show the approach from each side.

Example:

Given $f(x) = |x-2|$. Is $f(x)$ Continuous at $x = 2$? Prove it.

(1)

$f(2) = 0$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} -(x-2) = 0 = \ell_1$$

and

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-2) = 0 = \ell_2$$

and

$$\ell_1 = \ell_2 = 0$$

(2)

$$\lim_{x\to a} f(2) = 0$$

(3)

$0 = 0$

So $f(x)$ is continuous at $x = 2$.

Thm: If a function is differentiable, then the function is continuous.

Proof:

Assume: $\lim_{x\to a} f(x) = f(a)$ is true.

$$\lim_{x\to a} f(x) - f(a) = 0$$

$$\lim_{x\to a} f(x) - f(a)\cdot\frac{(x-a)}{(x-a)} = 0$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{(x-a)}\cdot(x-a) = 0$$

$$\underbrace{\lim_{x\to a} \frac{f(x)-f(a)}{(x-a)}}\cdot(x-a) = 0$$

$$f'(a)\cdot 0 \qquad = 0$$

$\uparrow$

given

as existing

Is this an example of "assuming what we are trying to prove? Yes and it is allowable as long as we understand the significance of the possible results.

If we end with a "truth condition" then we have, in fact, proved the theorem.

If we end with a "false condition" then we have proven nothing. That is we have neither proved the theorem to be true or false.

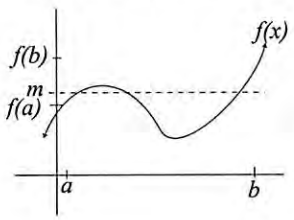
Thus, if $f'(a)$, ie. if f has a derivative at $x = a$, then f is continuous at $x = a$.

The Intermediate Value Theorem

Let f be continuous in [a, b]. Let m be a number such that $f(a) \le m \le f(b)$ (or vice-versa) then there exists at X such that $f(X) = m$.

The car speed is 25 mph and then later it is 50 mph. Did it ever read 40 mph? Of course. How do you know? We know because the speed changes are continuous so to get from 25 to 50, it had to read 30 and 35 as well as 40mph.

Graphically:



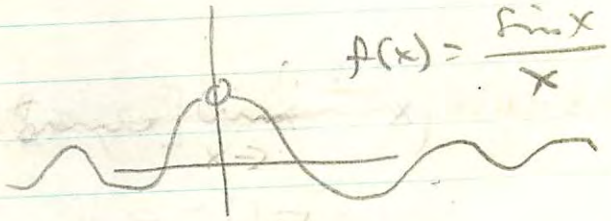
Note: m is a range item which is between $f(a)$ and $f(b)$.
In this example, there are three values which could be ~~X~~ a result of F(X)

We used the IVT in Analysis. We found $f(2) = 5$ and $f(3) = -4$ so we knew there was a root between 2 and 3. ie. $m = 0$, $a = 2, b = 3$, and X might have been $\frac{11}{5}$.

It must be time to enjoy.

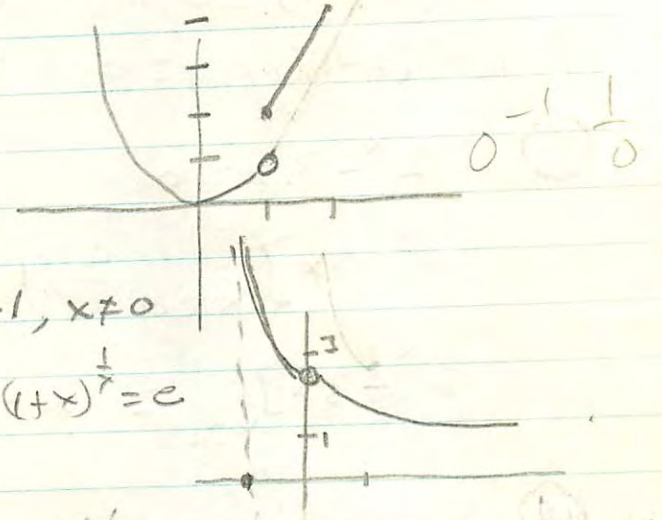
4.7
Pg 95-96

(1) Continuous $[0,1]$
(2) Not continuous $0, \frac{1}{4}, \frac{1}{2}$
(3) Not continuous $\frac{1}{2}$
(4) Continuous $[0,1]$
(5) Not cont. 1
(6) Not cont 0
(7) Yes define $f(0)=1$



(8) $f(x) = \begin{cases} 2x; x>1 \\ x^2; x<1 \end{cases}$

NO



(9) $f(x) = (1+x)^{\frac{1}{x}}$ $x>-1, x\neq 0$
(graph pg 76)

$\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

a) Yes let $f(0) = e$

b) No because as $x \to -1$ $f(x)$ continues toward ∞

(10) $f(x) = \frac{x^3-1}{x-1}$ $x\neq 1$ x^2+x+1

$\lim_{x\to 1} \frac{x^3-1}{x-1} = 3$ ∴ define $f(1)$ the 3.

(11) $f(x) = 3x+5$ $[1,2]$ $m=10$ $3x+5 \overset{\text{set}}{=} 10$
$x = \frac{5}{3}$

(12) x^2-2x $[-1,4]$ $m=5$ $x^2-2x \overset{\text{set}}{=} 5$
$x = \frac{2 \pm \sqrt{4+20}}{2} \Rightarrow x = \frac{2 \pm 2\sqrt{6}}{2}$ $1 \pm \sqrt{6}$ only $1+\sqrt{6}$
$1-\sqrt{6} < -1$

(13) $\sin x$ $\left[\frac{\pi}{2}, \frac{11\pi}{2}\right]$ $m=0$

$\pi, 2\pi, 3\pi, 4\pi, 5\pi$

(14) $\cos x \quad [0, 5\pi] \quad m = 0 \quad \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

(15) $\cos x \quad [0, 5\pi] \quad m = \pm\frac{1}{2}$

(16) $2^x \quad [0, 3] \quad m = 4$

$2^x \overset{\text{set}}{=} 4$

$2^x = 2^2$

$x = 2$

(17) $x^3 - x \quad [-2, 2] \quad m = 0$

$x(x^2 - 1) = 0$

$(x)(x-1)(x+1) = 0$ $\quad \{-1, 0, 1\}$

(18) $2^x - 3x = 0 \quad [0, 1]$

$2^x - 3x\Big]_{x=0} = 1 - 0 = 1$

$2^x - 3x\Big]_{x=1} = 2 - 3 = -1$

change of sign: 1 solution in $[0, 1]$

(19)

$x + \sin x = 1 \quad [0, \frac{\pi}{2}]$

$\sin x + x - 1 = 0$

$f(x) = \sin x + x - 1$

$f(0) = 0 + 0 - 1 = -1$

$f(\frac{\pi}{2}) = 1 + \frac{\pi}{2} - 1 = \frac{\pi}{2}$

(20) $x^3 = 2^x \quad [1, 2]$

$f(x) = x^3 - 2^x$

$f(1) = 1 - 2 = -1$

$f(2) = 8 - 4 = 4$

(21) $3x^3+11x^2-5x=2$

$3x^3+11x^2-5x-2=0$

	3	11	−5	−2
0				−2
1	3	14	9	7

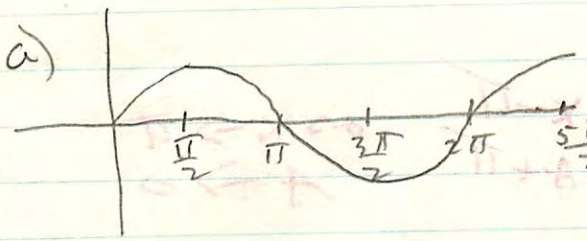
(22) a) No. Consider $y=|x|$

b) Yes. If it has a derivative at a then it must be continuous at a (so there is some place to have the derivative)

(23) Yes since f is continuous $\forall x$

(24) $f(x)=\sin x \quad a=0 \quad b=\frac{5\pi}{2}$

a)



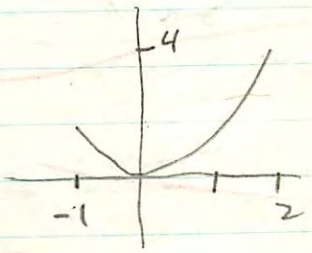
b) $0, \pi, 2\pi$

c) 3, for $m=0$
2 for $m=1$

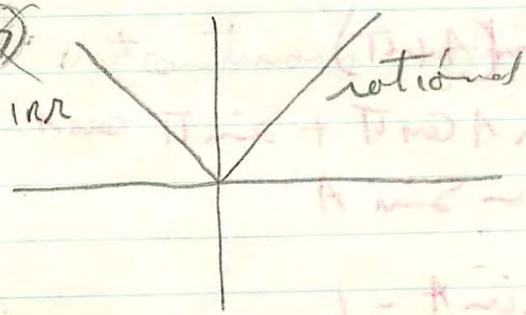
~~(25)~~ $f(x)=x^2 \quad a=-1 \quad b=2$

for $1<m<4$ $\underline{X}=\sqrt{m}$

for $|m|\leq 1$ $\underline{X}=\pm\sqrt{m}$



~~(27)~~



continuous at $x=0$ only.

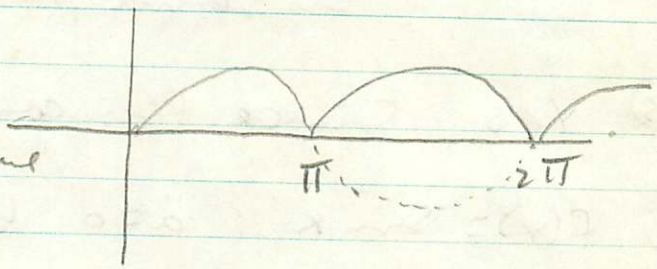
(28) $f(x) = \frac{1}{x}$ $a = -1$ $b = 1$ $m = 0$

No. This does not invalidate the intermediate value thm because f is not continuous through the interval $[-1, 1]$.

~~(29)~~ $f(x+y) = f(x) + f(y)$

a) $f(x+x) = f(x) + f(x) =$
$f(2x) = 2f(x)$

(30) $f(x) = |\sin x|$



a) continuous everywhere
b) No derivative at
$x = k\pi$.

$\lim_{x \to \pi} \frac{|\sin x|}{x - \pi k}$ Let $A = x - \pi$ $x = A + \pi$ as $x \to \pi$ $A \to 0$

~~$\lim_{x \to \pi^-} \frac{\sin x}{\sin x} = 1$~~

~~$\lim_{x \to \pi^+} \frac{-\sin x}{\sin x} = -1$~~

$\therefore$ der DNE

$\lim$

$\sin x = \sin(A + \pi)$

$\sin A \cos \pi + \sin \pi \cos A$

$-\sin A$

$\lim_{A \to 0} \frac{|-\sin A|}{k A \pi}$

$\lim_{A \to 0^-} \frac{-\sin A}{A} = -1$ $\lim_{A \to 0^+} \frac{+\sin A}{A} = 1$

(L) $\neq$ (R)

Guide Quiz on exponents.

(1) a) $8^{-1} = \frac{1}{8} = .125$ b) $8^{1/2} = 2\sqrt{2} \approx 2.83$

c) $8^{2/3} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 4$ D) $8^0 = 1$ e) $8^{-1/2} \approx .354$

f) $\frac{1}{4} = .25$

(2) a) $\sqrt{3} = 3^{1/2}$ b) $\frac{1}{3} = 3^{-1}$ c) $\sqrt[3]{9} = 3^{2/3}$ d) $\frac{3^7}{3^2} = 3^5$

e) $3\sqrt{3} = 3^{3/2}$ f) $(3^5)^7 = 3^{35}$ g) $\frac{1}{\sqrt{3}} = 3^{-1/2}$ h) $3^2 \cdot 9^4 =$
$3^2 \cdot 3^8 = 3^{10}$

(3) $7^{1/3}$

$7^{1/3} < 8^{1/3}$

$7^{1/3} = (1.9)$

$1.9 \cdot 1.9 = 3.61$

$3.61 \cdot 1.9 = 6.859$

$\frac{8}{6} = 4$

(4) a) $(6^{1/2})^4 = 6^2$ b) $6\sqrt{6} = 6^{3/2}$ c) $\sqrt{6} \cdot \sqrt[3]{6} = 6^{\frac{1}{2}+\frac{1}{3}} =$
$6^{5/6}$

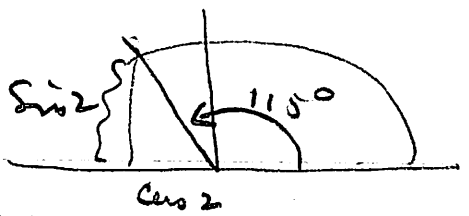
d) $\left(\frac{1}{\sqrt{6}}\right)^3 = 6^{-\frac{3}{2}}$ e) $\frac{6\sqrt{6}}{\sqrt[3]{6^2}} = 6^{3/2 - 2/3} = 6^{5/6}$

f) $\left(\sqrt{6^3}\right)^5 = 6^{\frac{3}{2} \cdot 5} = 6^{\frac{15}{2}}$

(5) a) $\left(\sqrt[3]{5}\right)^{12} = 5^4 = 625$ $\left(\sqrt[4]{5}\right)^{12} = 5^3 = 125$

b) $\sqrt[3]{5} \overset{?}{<} \sqrt[4]{5}$ True.

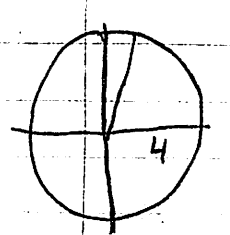
Guide Quiz TRig.

1. a) $2 \text{ rad} \approx 114.6$ degrees. b) 

$\pi \text{ rad} \Leftrightarrow 180$ deg.

$1 \text{ rad} = \frac{180}{\pi}$

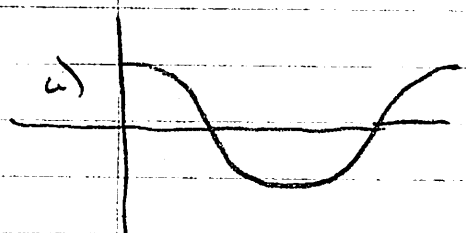
$2 \text{ rad} = \frac{360}{\pi}$

2. 

$1.3 \text{ Rad} \approx 74.5°$

a) arc length. $4(1.3) = 5.2$

b) $\frac{\theta R^2}{2}$ $\frac{(1.3)(16)}{2} - (1.3)8 = 10.4$

3. a)  $y = \cos\theta$ b) Same curve as $\sin\theta$ out of phase.

Guide Quiz Chapt 4

1 a) $e = \lim_{N\to\infty}\left(1+\frac{1}{N}\right)^N = \lim_{x\to 0}(1+x)^{\frac{1}{x}} = \lim_{N\to-\infty}\left(1+\frac{1}{N}\right)^N$

b) $e = 2.718$

c) $\lim_{h\to 0}\left(1+\frac{h}{3}\right)^{\frac{3}{h}} = \lim_{h\to 0}\left(1+\frac{1}{\frac{3}{h}}\right)^{\frac{3}{h}} = e$

2. $c^5 - d^5 = (c-d)(c^4 + c^3d + c^2d^2 + cd^3 + d^4)$

$(c-d)(c^4+c^4+c^4+c^4+c^4) > c^5 - d^5$

$5c^4(c-d) > c^5 - d^5$

3. a) 2 b) None c) None d) 2 e) $-1, 0, 2, 4, 5$

4. a) $\lim_{x\to\infty}\frac{5x^3-6x+2}{10x^3+5} = \lim_{x\to\infty}\frac{5-\frac{6}{x^2}+\frac{2}{x^3}}{10+\frac{5}{x^3}} = \frac{1}{2}$

b) $\lim_{x\to 0}\frac{\tan 3x}{\tan 2x} = \lim_{x\to 0}\frac{\sin 3x}{\cos 3x}\cdot\frac{\cos 2x}{\sin 2x}\cdot\frac{3x}{3x}\cdot\frac{2x}{2x} =$

$\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{2x}{\sin 2x}\cdot\frac{3}{2}\cdot\frac{\cos 2x}{\cos 3x}$

$\Downarrow \quad \Downarrow \quad \Downarrow \quad \frac{1}{1} = \frac{3}{2}$

$1 \quad 1 \quad \frac{3}{2}$

c) $\lim_{x\to 1}\frac{3\cos x - 3}{x} = 3\cos 1 - 3 \approx -1.38$

d) $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{1}{\sqrt{x}+2} = \frac{1}{4}$

e) $\lim_{x\to 1^+} 2^{\frac{1}{x-1}}$ No limit as $x\to 1^+$ $\frac{1}{x-1}\to+\infty$

f) $\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)^{3x} = \lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right)^3 = e^3$

(5) D. $\lim_{x\to 4}\frac{f(x)-f(a)}{x-a}$ where $f(x) = \sqrt{x}$, $a = 4$.

98-99

6. $\frac{\overline{AP}}{\overline{BP}} \to 1$ $\qquad$ $\frac{\overline{AB}}{\overline{BP}} \to 0$

$\Downarrow$ $\qquad$ $\Downarrow$

$\lim_{x\to 0} \frac{\sin x}{x}$ $\qquad$ $\lim_{x\to 0} \frac{1-\cos x}{x}$

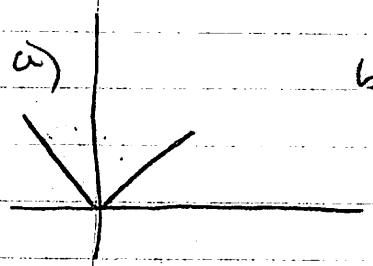
7. $V = 2^{-\tau} \sin^2 \pi\tau$

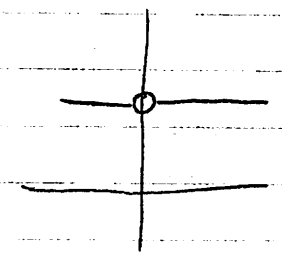
a) $\tau = 0 \quad V = 0 \qquad \tau = \frac{1}{2} \quad 2^{-\frac{1}{2}} \sin^2 \frac{\pi}{2} = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}}$

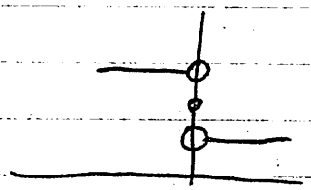
b) intermediate value thm $m = \frac{1}{2}$ and

$0 < \frac{1}{2} < \frac{1}{\sqrt{2}}$

8.

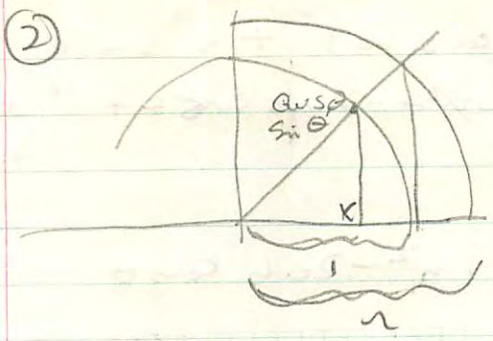
a) 

b) 

c) 

Review ex chapt 4

(2)

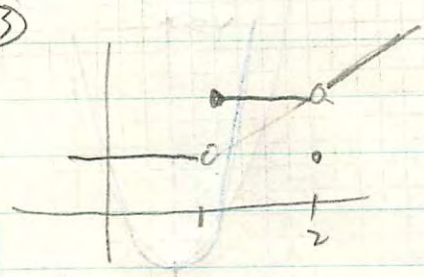


Δs similar Δ AA

$\frac{\cos\theta}{1} = \frac{x}{r}$ $\therefore$ $x = r\cos\theta$

etc.

(3)



(4)

a) circum $2\pi r$

b) area πr^2

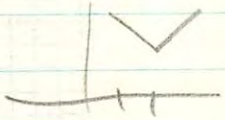
c) area sector $\frac{\theta R^2}{2}$

D) arc length also θr

(5) a false

b) True

d) True



(6)

(7)

a) $\lim_{h\to 0} \frac{1-\cos h}{h^2} \cdot \frac{1+\cos h}{1+\cos h} = \lim_{h\to 0} \frac{1-\cos^2 h}{h^2(1+\cos h)} =$

$\lim_{h\to 0} \frac{\sin^2 h}{h^2} \cdot \frac{1}{1+\cos h}$ =

$\sin h \to h$: $1 \cdot 1$ $\quad$ $\cos h \to 1$: $\frac{1}{2}$ $= \frac{1}{2}$

c) π rad $= 180$

1 rad $= \frac{180}{\pi}$

Sub $\left(\frac{\pi}{180}\right) h$ for h.

$\cos h\left(\frac{\pi}{180}\right) = 1 - \frac{\left(\frac{\pi}{180}\right)^2 h^2}{2}$

$= 1 - .00015\, h^2$

b) $\frac{1-\cos h}{h^2} = \frac{1}{2}$

$1 - \cos h = \frac{h^2}{2}$

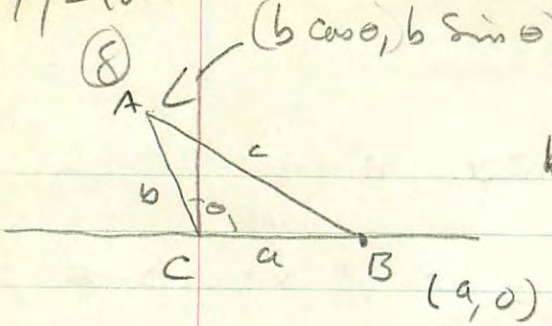
$\cos h = 1 - \frac{h^2}{2}$

d)

10°	.9848	.984769	.0001
5°	.99619	.9962	.00001
1°	.99847	.999847	—

99-100

(8)



b) $c^2 = (b\cos\theta - a)^2 + b^2\sin^2\theta$

$= b^2\cos^2\theta - 2ab\cos\theta + a^2 + b^2\sin^2\theta$

$+ b^2 - b^2\cos^2\theta.$

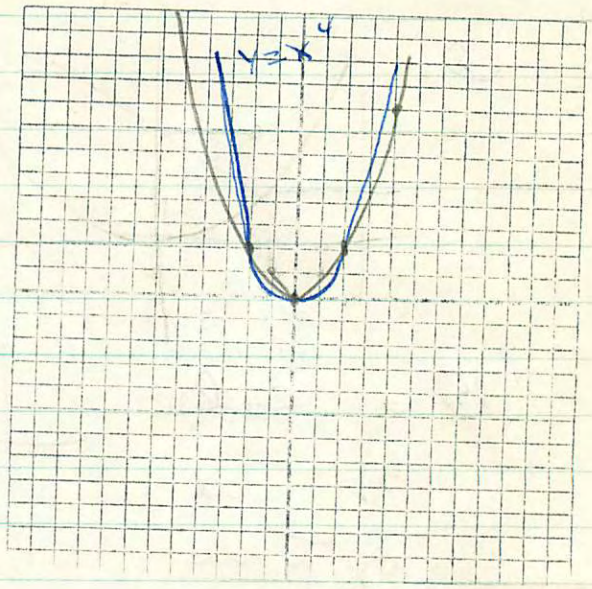
$c^2 = a^2 + b^2 - 2ab\cos\theta.$

9. continuous at intersections of $Y = x^2$ & $Y = x^4$

so $0, \pm 1$

Differentiable only at zero

(2)



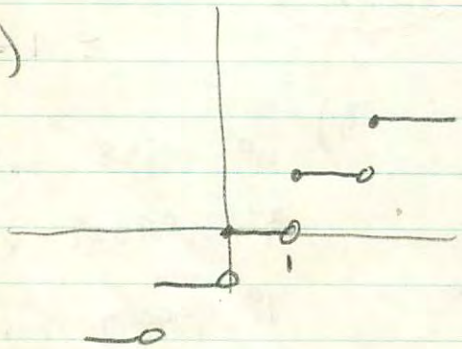
10.

11. a) $+\infty$ b) $-\infty$ c) by intermediate Value Thm $-\infty < 0 < \infty$

12. a) See 11 b) $x^2 + 1 = 0$

13.

a)



b) Not cont at $a \in J$.

14. $\tan x = 3 + \cos^2 x$

$\tan x - 3 - \cos^2 x = 0$

$x = 0 \quad 1 - 3 - 1 = -3$

$x \Rightarrow \frac{\pi}{2} \quad \text{huge} - 3 - 0 = +\infty$

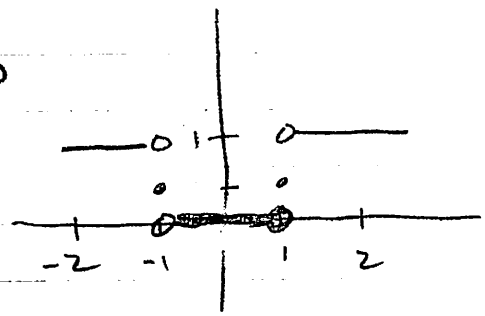
15. $f(x) = \lim_{N\to\infty} \frac{x^{2N}}{1+x^{2N}}$

b) ALL $x \in D_f$

$f(\tfrac{1}{2}) = \lim_{N\to\infty} \frac{(\frac{1}{2})^{2N}}{1+\frac{1}{2}^{2N}} = \frac{0}{1+0} = 0$

$f(2) = \lim_{N\to\infty} \frac{2^{2N}}{1+2^{2N}} = 1$

$f(1) = \lim_{N\to 0} \frac{1^{2N}}{1+1^{2N}} = \frac{1}{2}$

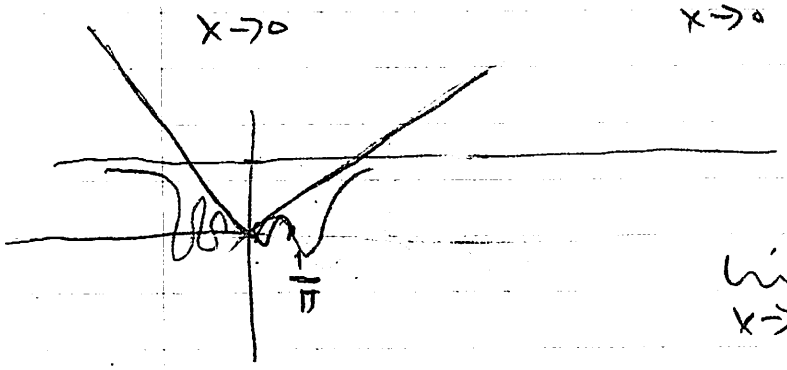


i) continuous $|x|>1$
$|x|<1$

(18) $f(x) = x \sin\frac{1}{x}$

$\lim_{x\to 0} x \sin\frac{1}{x} = \lim_{x\to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$

when $x \to 0$
$\sin\frac{1}{x} < 1$
but $\frac{1}{x}$ is large
so $\lim_{x\to 0} x\sin\frac{1}{x} = 0$



$\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$

because $\frac{1}{x} \to 0$

near zero $\frac{1}{x}$ is large
$\therefore \sin\frac{1}{x}$ will take on + then − values ie oscillate

also $\lim_{x\to -\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1$

when $|x|>1$ the curve will start approaching $y=1$

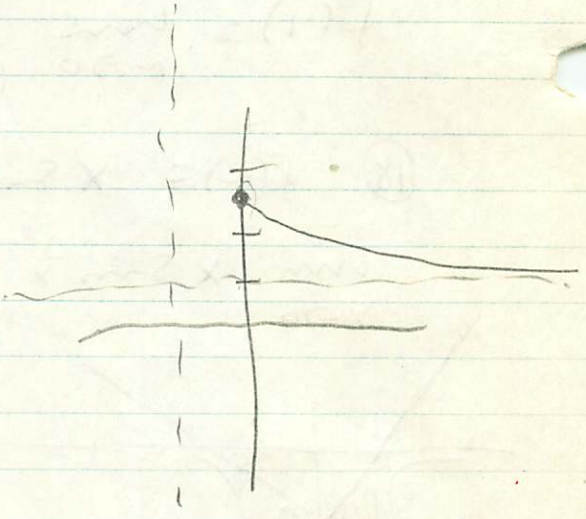
(19) $f(f(f(x))) = x$

$f(\bigstar) = x$

$\therefore \bigstar = f(f(x)) = x$

#6 pg 100 $f(x) = (1+x)^{\frac{1}{x}}$

$\lim_{x \to 0} f(x) = e$



Review CHAPT 1-4

1. a) Derivative of a fcn at x is the slope of the line tangent at x to the curve.
   b) slope, rate, density, magnifications

2. a) $\left.\frac{d(2x^4)}{dx}\right]_{x=\frac{1}{2}} = \left.8x^3\right]_{x=\frac{1}{2}} = 1$

   b) $\left.\frac{d(x^3+5x)}{dx}\right]_{x=-1} = 3x^2+5 = 8$

   c) $\left.\frac{d(\sqrt{x})}{dx}\right]_{x=4} = \left.\frac{1}{2\sqrt{x}}\right]_{x=4} = \frac{1}{4}$

   d) $\left.\frac{d\left(\frac{1}{x}\right)}{dx}\right]_{x=\sqrt{2}} = \left.-\frac{1}{x^2}\right]_{x=\sqrt{2}} = -\frac{1}{2}$

3. $f(x) = x^3$

   a) $f(x+h)-f(x)$ $\quad f(2.5)-f(2) = 15.625-8 = 7.625$

   b) $f(\tau_1)-f(\tau)$ $\quad f(3.1)-f(3) = 29.791-27 = 2.791$

4. $h = 3\sin\tau$

   $h' = 3\cos\tau\,]_{\tau=0} = 3$

5. a) $(-1,4)\ (2,6)$ $\quad \frac{6-4}{2+1} = \frac{2}{3}$

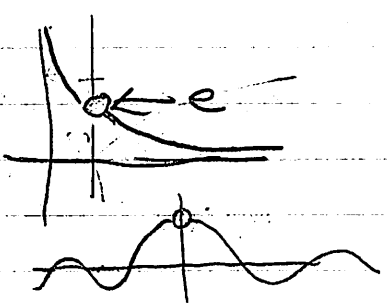
   b) $6 = m$

6. $f(x) = (1+x)^{\frac{1}{x}}$ $\quad D_f\ x>-1\ x\neq 0$

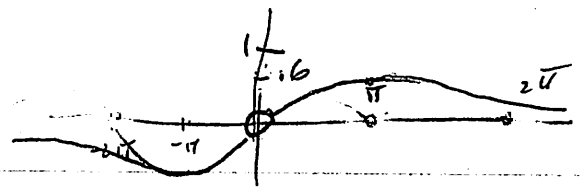
   $(1+x)^{\frac{1}{x}} = \sqrt[x]{1+x}$

   $\therefore x > -1$

7. $f(x) = \frac{\sin x}{x}$ $\quad D_f\ x\in\mathbb{R}\ x\neq 0$



100-101



(8) $f(x) = \frac{1-\cos x}{x}$

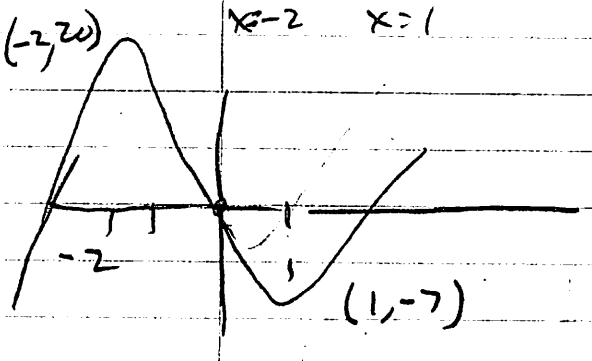
D_f $x \in \mathbb{R}$ $x \neq 0$.

(9) $f(x) = 2x^3+3x^2-12x$ a) $f'(x) = 6x^2+6x-12$

b) $x^2+x-2=0$ zero $x=-2, 1$ + $x>1$ or $x<-2$

$(x+2)(x-1)=0$ − $-2<x<1$

$x=-2$ $x=1$



	2	3	-12	0
-2	2	-1	-10	20
1	2	5	-7	-7

(10) a) $\lim_{h\to 0} \frac{(x+h)^4+5(x+h)^3-x^4-5x^3}{h} =$

$\lim_{h\to 0} \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4+5x^3+15x^2h+15xh^2+5h^3 - x^4-5x^3}{h}$

$\lim_{h\to 0} 4x^3+6x^2h+4xh^2+h^3+15x^2+15xh+5h^2 = 4x^3+15x^2$

b) $\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{x^2-x^2-2xh+h^2}{x^2h(x+h)^2} = \frac{-2+h^2}{x(x+h)^2} =$

(11) a) Slope of graph of f is 3 at $x=1$ $\frac{-2}{x^3}$

b) Speed is 3 ft/sec, in $\frac{1}{100}$ of sec it travels .03 feet,

c) magnification is 3 times, length is .03

d) Density at $x=1$ is 3 grams/centimeter, .03

(13) a) $6x^3 - 5x$ $\quad 1 + x^2 - 5$

b) $(1+3x^2)^2$ $\quad 1 + 6x^2 + 9x^4$ $\quad 1 + 12x + 36x^3$

c) $\frac{5}{x}$ $\quad -\frac{5}{x^2}$

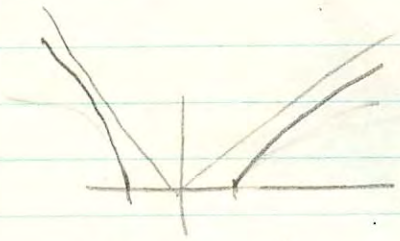
(14) $$\lim_{h\to 0} \frac{\frac{(x+h)^2}{(1+x+h)} - \frac{x^2}{1+x}}{h} = \lim_{h\to 0} \frac{(x^2+2xh+h^2)(1+x) - x^2 - x^3 - x^2h}{h(1+x)(1+x+h)}$$

$x^2 + 2xh + h^2 + x^3 + 2x^2h + xh^2$

$-x^2 \qquad\qquad -x^3 - x^2h$

$$\lim_{h\to 0} \frac{2xh + h^2 + x^2h + xh^2}{h(1+x)(1+x+h)} = \lim_{h\to 0} \frac{2x + h + x^2 + xh^2}{(1+x)(1+x+h)} =$$

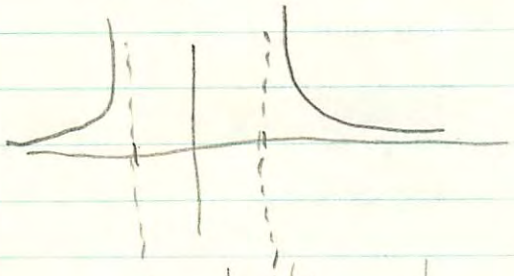
$$\frac{x^2+2x}{(1+x)^2}$$



Yes, continuous $\forall x \in D_f$.

(15) $f(x) = \sqrt{x^2-1}$

$D_f \quad |x| \geq 1$

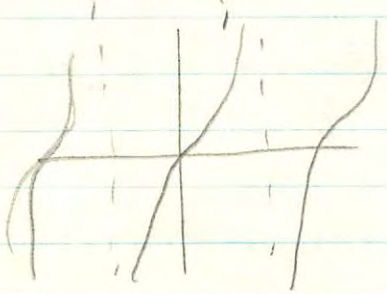


(16) $f(x) = \frac{1}{\sqrt{x^2-1}}$

$D_f \quad |x| > 1$

(17) $f(x) = \tan x$

$x \in \mathbb{R} \quad x \neq \left\{ \frac{2k+1}{2}\pi \right\}$



(18) a) $\lim_{x\to 0^+} 2^{\frac{x}{|x|}} = 2$

b) $\lim_{x\to 0^-} 2^{\frac{x}{|x|}} = \frac{1}{2}$

c) $\lim_{x\to 1} 2^{\frac{x}{|x|}} = 2$

(19) a) $\lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{4x+2}} = \sqrt{\frac{x}{4x+2}} = \sqrt{\frac{1}{4+\frac{2}{x}}} = \frac{1}{2}$

b) $\lim_{x\to\infty} \frac{8x^4+6x-2}{2x^4-5x} = \frac{8+\frac{6}{x^3}-\frac{2}{x^4}}{2-\frac{5}{x^3}} = 4$

c) $\lim_{x\to\infty} \left(\frac{5x^2+4x}{3x+1} - \frac{5x+8}{3}\right) =$

$\lim_{x\to\infty} \frac{15x^2+12x-15x^2-24x+8}{3(3x-1)} =$

$\lim_{x\to\infty} \frac{-17x+8}{9x-3} = \frac{-17}{9}$

(20)

a) $f'(x)>0$ f is increasing.
$f'(x)<0$ f is decreasing

b) $f'(x)>0$ Particle is moving up
$f'(x)<0$ Particle is moving down.

(21) $\lim_{x\to 0} \frac{\tan x - \sin x}{x} = \lim_{x\to 0} -\frac{\sin x}{x} + \frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$-1 \quad +1\cdot\frac{1}{1} = 0$

(22) $\lim_{x\to 0} \frac{\tan x - \sin x}{x^2} = \lim_{x\to 0} \frac{\sin x}{x}\left(\frac{1-\cos x}{x}\right)\cdot\frac{1}{\cos x}$

$1 \qquad 0\cdot(=0$

(23) a) $\lim_{x\to 0^+} \frac{2^{\frac{1}{x}}+1}{2^{\frac{1}{x}}+3} = \lim_{a\to+\infty} \frac{2^a+1}{2^a+3} = 1$ ($a\to+\infty$, $2^a\to\infty$)

b) $\lim_{x\to 0^-} \frac{2^{\frac{1}{x}}+1}{2^{\frac{1}{x}}+3} = \lim_{a\to 0^+} \frac{\frac{1}{2^a}+1}{\frac{1}{2^a}+3} = \lim_{b\to\infty} \frac{2^b+1}{\frac{1}{2^b}+3} = \frac{1}{3}$ ($a\to 0^+$, $\frac{1}{a}\to\infty$, $b\to\infty$, $2^b\to\infty$, $\frac{1}{2^b}\to 0$)

(24) $f(x) = e^x$

$$\lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x(e^h - 1)}{h}\Bigg]_{x=0} = \lim_{h\to 0} \frac{e^h - 1}{h}$$

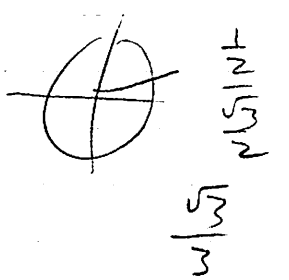
h	.15	.1	.05	.01
$\frac{e^h-1}{h}$	1.0789	1.05171	1.0254	1.00502

The limit appears to be 1.

(25) a) x^3 b) $\frac{x^3}{3}$ c) $\frac{1}{x}$ d) $-\frac{1}{x}$

e) $\frac{9}{4}x^4 + 2x^3 - 3x^2 + 2x + C$ f) $\frac{5\sqrt{x}}{2}$

(26) $$\lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h} =$$



$$\lim_{h\to 0} \frac{\frac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x}{h} =$$

$$\lim_{h\to 0} \frac{\tan x + \tan h - \tan x + \tan^2 x \tan h}{h(1 - \tan x \tan h)}$$

$$\lim_{h\to 0} \frac{\tan h(1 + \tan^2 x)}{h(1 - \tan x \tan h)} = \lim_{h\to 0} \frac{\sin h}{h} \cdot \frac{1 + \tan^2 x}{\cos h(1 - \tan x \tan h)}\Bigg]_{x=\frac{\pi}{6}}$$

$$\frac{1 + \frac{1}{3}}{1(1-0)} = \frac{4}{3}$$

(27) $\lim_{h \to 0} \dfrac{\sqrt[3]{x+h} - \sqrt{x}}{h}$ routine.

recognize as $f(x) = \sqrt[3]{x}$ and find $f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$

(28)

(29) assume $f(x) > x \quad \forall x \in [0, 1]$

$x = f(f(x)) > f(x) > x$

(30)

(31) a) sub stuff in $A = P\left(1 + \frac{1}{N}\right)^{Nt}$

b) $\lim_{N\to\infty} \left(1 + \frac{.05}{N}\right)^{Nt} = e^{.05t}$

$\lim_{N\to\infty} \left(\left(1 + \frac{1}{20N}\right)^{20N}\right)^{\frac{t}{20}} = e^{\frac{t}{20}} = e^{.05t}$

c) ~~$2 = \left(1 + \frac{1}{20N}\right)^{Nt}$~~ $e^{.05t} = 2$ solve for t

$e^{\frac{t}{20}} = 2$

$\frac{t}{20} = \ln 2 \quad t = 20 \ln 2$

$t \approx 13.86$ yrs.

(32) $\lim_{x\to\infty} \left(4^x + 3^x\right)^{\frac{1}{x}}$ approaches $\to 4$,

$4^x < 4^x + 3^x \leq \frac{4^x + 4^x}{2 \cdot 4^x}$

$2 \cdot 2^{2x}$

2^{2x+1}

$\left(2^{2x+1}\right)^{\frac{1}{x}} = 2^{2 + \frac{1}{x}}$

$\lim_{x\to\infty} \left(4^x\right)^{\frac{1}{x}}$

$\lim_{x\to\infty} \left[2^{2+\frac{1}{x}}\right]^x = 4$ but $\lim_{x\to\infty} \left(4^x + 3^x\right)^{\frac{1}{x}} = z$.

4

since $4 \leq z \leq 4$

$z = 4$

(33) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \frac{3}{2}$ 101-102

(34) a) $N = m$ $\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \frac{a_0}{b_0}$

b) $N < m$ $\lim \to 0$

c) $N > m$ $\to \pm\infty$ depending if $\frac{a_0}{b_0} > 0$ or < 0.

(35) No The curve oscilates between the two lines $y = x$ & $y = -x$

36. $\lim_{x\to\infty} (\sqrt{x^2+2x} - x) \frac{\sqrt{\quad}+x}{\sqrt{\quad}+x} = \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x} =$

$\lim_{x\to\infty} \frac{2x}{x\sqrt{1+\frac{2}{x}}+x} = \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = \frac{2}{2} = 1$

the arguement that $\sqrt{x^2+2x} \Rightarrow x$ $\forall\, x\to\infty$ is fine however $\infty - \infty$ is not defined. The subtraction, factoring etc must be done before you let $x\to\infty$.

37. $\lim_{x\to\infty} \left(\frac{3x^2+2x}{x+5} - 3x\right)$ Neither citizen is correct.

$\lim_{x\to\infty} \frac{3x^2+2x-3x^2-15x}{x+5}$

$\lim_{x\to\infty} \frac{-13x}{x+5} = \lim_{x\to\infty} \frac{-13}{1+\frac{5}{x}} = -13$

(38) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3-x}}{x} \frac{\sqrt{\quad}+\sqrt{\quad}}{\sqrt{\quad}+\sqrt{\quad}}$

$\lim_{x\to 0} \frac{3+x-3+x}{x(\sqrt{\quad}+\sqrt{\quad})} = \lim_{x\to 0} \frac{2}{\sqrt{\quad}+\sqrt{\quad}} = \frac{1}{\sqrt{3}}$

"Continuous Enjoyment -- 2nd Edition"

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

1) If $\sec x$ is positive then it can be expressed in terms of $\sin x$ as

a) $\frac{1}{\sin x}$ b) $\frac{1}{1-\sin^2 x}$ c) $\frac{1}{\sqrt{1-\sin^2 x}}$ d) $\frac{\sin x}{1-\sin x}$ e) $\frac{1}{1-\sin x}$

2) Which of the following functions is even?

I. $f(x)=\sin(x^2-1)$ II. $g(x)=\cos(x^3-2)$ III. $h(x)=(x-3)^2$

a) I only b) II only c) III only d) II and III only e) I, II, and III

3) Which of the following functions has a vertical asymptote at $x=\frac{\pi}{2}$?

I. $f(x)=\sin\left(x-\frac{\pi}{2}\right)$ II. $g(x)=\tan x$ III. $h(x)=\cot x$

a) I only b) II only c) III only d) II and III only e) I and III only

4) If the angle from the x-axis to a given line is 30°, then the slope of the line is:

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{3}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) 1

5) The period of the function $y=3\tan\left(\frac{2}{3}x-\pi\right)$ is

a) $\frac{2\pi}{3}$ b) π c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$ e) 3π

6) If $f(x)=\frac{\sin x-x}{1-\cos x}$ then $f(-x)=$

a) $\frac{\sin x+x}{1+\cos x}$ b) $\frac{\sin x-x}{1-\cos x}$ c) $\frac{x-\sin x}{1-\cos x}$ d) $\frac{-\sin x-x}{1-\cos x}$ e) $\frac{\sin x-x}{1+\cos x}$

7) Which of the following functions is neither even nor odd?

a) $f(x)=\tan x$ b) $f(x)=\sin^2 x$ c) $f(x)=\cos|x|$ d) $f(x)=\frac{1-\cos x}{\sin x}$ e) $f(x)=\sec x+\csc x$

8) If $f(x)=x^2$ then $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=$

a) a^2 b) $2a$ c) a^2+2a d) a^2-2a e) 0

9) The equation of the tangent line to $f(x)=-\frac{3}{x}$ at the point where $x=3$ is:

a) $y=\frac{1}{3}x-2$ b) $y=-\frac{1}{3}x$ c) $y=\frac{1}{3}$ d) $y=-3x+8$ e) $y=-3x+10$

10) $\lim_{x\to 1}\frac{x-1}{x^2+2x-3}=$

a) 0 b) no limit c) ∞ d) $\frac{1}{2}$ e) $\frac{1}{4}$

11) If $f(x) = 2 - \left|4 + \frac{x}{2}\right|$ then $f(x)$ is not differentiable at $x =$

a) -2 b) 2 c) 0 d) -8 e) 8

$\lim_{x \to -\frac{1}{3}} \frac{9x^2 - 1}{3x + 1} =$

a) 0 b) Nonexistent c) -1 d) -2 e) $-\frac{1}{3}$

13) $\lim_{x \to 2} \frac{\sqrt{x+14} - 4}{x - 2}$

a) 0 b) 8 c) $\frac{1}{8}$ d) 4 e) $\frac{1}{4}$

14) Let $f(x) = \frac{x - 7}{\sqrt{x-3} - 2}$ which of the following statements are true?

I. $\lim_{x \to 7} f(x) = 4$ II. $f(7)$ is undefined. III. $\lim_{x \to 7} f(x) = f(7)$

a) I only b) II only c) III only d) I and III only e) I and II only

15) Given $f(x) = 3x - 5$ and $g(x) = 3x^3 - 5x^2 - 3x + 5$. Find $\lim_{x \to \frac{5}{3}} \frac{g(x)}{f(x)}$.

a) $\frac{34}{9}$ b) $\frac{16}{9}$ c) 0 d) $\frac{5}{3}$ e) $\frac{16}{3}$

16) If $f(x) = \frac{3x - 1}{2x}$ then $\lim_{x \to 0} f(x) =$

a) 0 b) ∞ c) -1 d) $-\frac{1}{2}$ e) DNE

17) Evaluate: $\lim_{x \to 5} \sqrt{\frac{x^2 - 9}{x - 3}}$

a) 8 b) $2\sqrt{2}$ c) 5 d) $\sqrt{2}$ e) 2

18) Given $f(x) = \begin{cases} 3x - 2 & \text{if } x \geq 5 \\ -x^2 + 3 & \text{if } 0 < x < 5 \\ -x^3 + 1 & \text{if } x \leq 0 \end{cases}$. Find $\lim_{x \to 5^-} f(x)$.

a) 13 b) 28 c) -22 d) -124 e) None of These

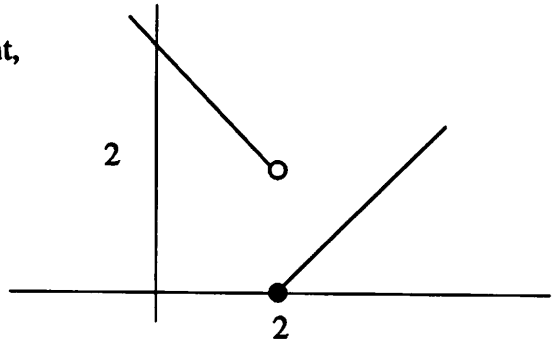
19) Given $f(x) = \frac{|2x - 4|}{x - 2}$. Find $\lim_{x \to 2^+} f(x)$.

a) DNE b) 0 c) 2 d) 1 e) 3

20) Given $f(x)=\begin{cases}\sqrt{(x^2-4)} & : \ x\le -2 \text{ or } x\ge 2\\ x^2-2 & ; \ -2<x<2\end{cases}$. Find $\lim_{x\to -2^-} f(x)$.

a) 2 b) -2 c) 4 d) 0 e) 1

21) From the graph of $f(x)$ shown on the right, which is necessarily true?



I. $\lim_{x\to 2^-} f(x)$ DNE

II. $\lim_{x\to 2^+} f(x)=0$

III. $\lim_{x\to 2} f(x)=0$

a) I only b) II only c) III only d) I and III only e) I and II only

22) Given $f(x)=\dfrac{x^2+2x}{x(x+2)^2(x-1)^4}$, then $\lim_{x\to a} f(x)=\infty$ for $a=$

a) $0,-2,1$ b) $0,1$ c) $1,-2$ d) $-1,2,0$ e) $-1,2$

23) The equation(s) of the vertical asymptote(s) of $f(x)=\dfrac{2x-5}{25-4x^2}$ is (are)

a) $y=\frac{5}{2}$ b) $x=-\frac{5}{2}$ c) $x=\frac{5}{2},\ x=-\frac{5}{2}$ d) $y=\frac{5}{2},\ y=-\frac{5}{2}$ e) $y=-\frac{5}{2}$

24) $\lim_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-25}$

a) $\frac{7}{10}$ b) DNE c) $\frac{2}{5}$ d) 1 e) 0

25) Which of the following functions has a vertical asymptote at $x=2$?

I. $f(x)=\dfrac{1}{x-2}$ II. $g(x)=\dfrac{x-2}{x-2}$ III. $h(x)=\begin{cases}\dfrac{1}{x-2} & ; \ x<2\\ x+5 & ; \ x\ge 2\end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

26) The equation of the horizontal asymptote of $f(x)=\dfrac{3x^2-2x+5}{7x-2x^2-1}$ is:

a) $y=\frac{3}{7}$ b) $x=\frac{3}{7}$ c) $y=-\frac{3}{2}$ d) $y=-5$ e) $x=-\frac{3}{2}$

27) Find $\lim_{x\to -\infty} \dfrac{x^2+4}{x^4-1}$.

a) 0 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) ∞ e) DNE

28) The equation of the horizontal asymptote to $xy - 2y = 3x - 4$ is:

a) $y = 2$ b) $x = 2$ c) $y = 3$ d) $x = 3$ e) none of these

29) $\lim_{x \to -\infty} \left(-5x + \frac{3}{2x^3}\right) =$

a) 0 b) ∞ c) $-\infty$ d) $\frac{3}{2}$ e) -5

30) The graph of the function $f(x) = \frac{x^2 + 2x}{x^2 - 2x}$ has asymptotes at:

a) $x = 0, x = 2, y = 1$ b) $x = 0, x = 2, y = 0$ c) $x = -2, y = 1$ d) $x = 0, x = -2, y = 1$ e) $x = 2, y = 1$

31) For what value(s) of x does $f(x) = \frac{x^2 - 2}{1 - x - x^2}$ cross its horizontal asymptote?

a) 0 b) $\pm\sqrt{2}$ c) -1 d) 1 e) -2

32) If $f(x) = \frac{x + 4}{2x - 1}$ which of the following values of x guarantees that $\left|f(x) - \frac{1}{2}\right| < \frac{1}{10}$?

a) $x < 20$ b) $x > \frac{46}{45}$ c) $x < 23$ d) $x > 23$ e) $x > 20$

33) Find $\lim_{x \to \infty} \frac{\sqrt{x^2 + 4x + 4}}{2x + 3}$

a) ∞ b) 0 c) $\frac{1}{2}$ d) $\frac{1}{3}$ e) 2

34) Given $f(x) = \begin{cases} 2 - 3x & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ 2x^2 - 5 & \text{if } x > 1 \end{cases}$. Which of the following statements are true?

I. $\lim_{x \to 1^-} f(x) = -1$ II. $f(1) = -1$ III. $\lim_{x \to 1} f(x) = f(1)$

a) I only b) II only c) III only d) I and II only e) I, II, III

35) Determine the value of $a + b$ so that the function $f(x) = \begin{cases} -3ax^2 - 1 & ; x > 1 \\ 3b - 2x & ; x \le 1 \end{cases}$ is continuous at $x = 1$.

a) $\frac{1}{3}$ b) 1 c) $-\frac{1}{3}$ d) -1 e) 0

36) Determine the value of k such that $f(x) = \begin{cases} 3kx - 5 & ; x > 2 \\ 4x - 5k & ; x \le 2 \end{cases}$ is continuous.

a) 3 b) $\frac{13}{11}$ c) $\frac{3}{11}$ d) $-\frac{3}{11}$ e) -3

37) Choose b so $f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & ; \ x<1 \text{ or } x>1 \\ b & x=1 \end{cases}$ is continuous.

a) $b \ \varepsilon \ \Re$ b) $b \ \varepsilon \ \{\ \}$ c) 3 d) 1 e) -1

38) Which of the following statements is **not** true?

a) If $f(x)$ is continuous at $x=a$, then $\lim\limits_{x\to a} f(x)$ exists.

b) If $f(x)$ is continuous at $x=a$, then it is defined at $x=a$.

c) If $f(x)$ is defined at $x=a$, then $f(x)$ is continuous at $x=a$.

d) If $\lim\limits_{x\to a} f(x) = f(a)$, then $f(x)$ is continuous at $x=a$

e) If $f(x)$ is a polynomial, then it is continuous at $x=a$.

add III to prblm 38

$f(+) = \{$

when f is NOT cont at $x=2$ but Df $x \in \mathbb{R}$.

39) Which of the following functions are continuous on $(-\infty, \infty)$?

I. $f(x) = x^2 + 4x - 3$ II. $f(x) = \dfrac{x+1}{x-1}$ III. $f(x) = \begin{cases} x^2+2x & ; \ x<2 \\ 2x+4 & ; \ x \geq 2 \end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

40) The function $f(x) = \begin{cases} -3x^2 - 1 & ; \ x > -2 \\ -13 & ; \ x = -2 \\ -x^2 - 17 & ; \ x < -2 \end{cases}$. Which of the following is true?

I. $\lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$ II. $\lim\limits_{x\to -2^-} f(x)$ DNE III. $f(-2) \neq \lim\limits_{x\to -2^-} f(x)$

a) I only b) II only c) III only d) I and III only e) I, II, III

41) $\lim\limits_{x\to 0} \dfrac{2\sin 3x}{5x} =$

a) $\dfrac{2}{5}$ b) $\dfrac{6}{5}$ c) 0 d) DNE e) $\dfrac{3}{5}$

42) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{2x - \frac{\pi}{2}}{\cos 2x} =$

a) 0 b) DNE c) $\dfrac{\pi}{4}$ d) -1 e) 1

43) $\lim\limits_{x\to \infty} \dfrac{\sin x}{2x} =$

a) 0 b) 1 c) ∞ d) $\dfrac{1}{2}$ e) 2

44) $\lim\limits_{x\to 0} \dfrac{\tan 2x}{x} =$

a) $\dfrac{1}{2}$ b) 0 c) ∞ d) 1 e) 2

45) $\lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x} =$

a) DNE b) -1 c) 0 d) ∞ e) 1

$\lim_{x \to 0} \left(\sin \frac{1}{x}\right)\left(\cos \frac{1}{x}\right) =$

a) 0 b) 1 c) $\frac{1}{2}$ d) -1 e) DNE

47) Which of the following functions is continuous and differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

48) Which of the following functions is differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

49) Find the value of $a + b$ such that f is differentiable at $x = -1$ if $f(x) = \begin{cases} 3x^2 & ; x < -1 \\ ax + b & ; x \geq -1 \end{cases}$.

a) 3 b) -9 c) -6 d) -3 e) 6

If $f(x) = \begin{cases} 3x^2 + 4 & ; x < 2 \\ bx^3 & ; x \geq 2 \end{cases}$, find b so that $f(x)$ is continuous.

a) 3 b) $\frac{1}{2}$ c) -1 d) 1 e) 2

51) If $f(x) = \begin{cases} 3x^2 + 4x & ; x \leq 1 \\ 2x^3 + bx + c & ; x > 1 \end{cases}$, find b and c so that $f(x)$ has a derivative at $x = 1$.

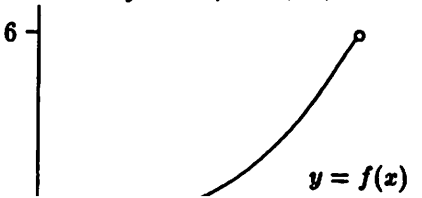
a) b=4, c=0 b) b=1, c=4 c) b=4, c=1 d) b=1, c=0 e) b=1, c=1

52) $f(x) = |x^3|$ then which of the following are true?

I. $f(x)$ is symmetric with respect to the origin.
II. $f(x)$ is continuous for all reals.
III. $f(x)$ is differentiable for all reals.

a) I only b) II only c) III only d) II and III only e) I, II, III

53) The domain of the function $f(x)$ graphed on the right is:



a) $[-2, 5]$ b) $[-3, 6)$ c) $[-2, \infty)$

d) $(-2, 5)$ e) $[-2, 6)$

54. A few graphing problems. Using MODE, select DEGREE and PAR. All other options should be set to left hand option. Using Y=, make $X_{1T} = 8 \sin t$ and $Y_{1T} = 8 \cos t$. Make the viewing window Tmin = 0, Tmax = 360, Tstep = 10, [–15, 15] for the x values and [–10, 10] for the y values.

a) Press GRAPH. Describe the picture you got.
b) Set Tstep = 1. Press GRAPH. Describe the picture you got.
c) Set Tstep = 20. Press GRAPH. Describe the picture you got.
d) Set Tstep = 40. Predict the picture you will get. Press GRAPH. Describe the picture you got.
e) Set Tstep = 60. Predict the picture you will get. Press GRAPH. Describe the picture you got.
f) Set Tstep = 72. Predict the picture you will get. Press GRAPH. Describe the picture you got.
g) Set Tstep = 120. Predict the picture you will get. Press GRAPH. Describe the picture you got.
h) Reset Tmax to 720. Predict the picture you will get. Set Tstep = 144. Press GRAPH. Describe the picture you got.
i) Reset Tmax to 30000. Predict the picture you will get. Set Tstep = 97. Press GRAPH. Describe the picture you got.
j) Summarize and state a conclusion.

55. A few graphing problems. Using MODE, select DEGREE and PAR. All other options should be set to left hand option. Using Y=, make $X_{1T} = 0.003t \sin t$ and $Y_{1T} = 0.003t \cos t$. Make the viewing window Tmin = 0, Tmax = 3000, Tstep = 6, [–15, 15] for the x values and [–10, 10] for the y values.

a) Press GRAPH. Describe the picture you got.
b) As in problem 54 above, select several more Tsteps which will yield interesting pictures. (Hint: You may have to change Tmax.)
c) Summarize and state a conclusion.

56. Write a calculator program which asks the user for two numbers and responds with the average of the two numbers.

57. Write a calculator program which asks the user for two numbers and responds with the square root of the product of the two numbers.

The following problems are intended only as "extras for experts."

*58. Write a calculator program which asks for the slope and y-intercept of a line and responds with the graph of the line.

*59. Write a calculator program which asks for the coordinates of two non-vertical points and responds with the graph of the line.

*60 Write a program which asks the user for the number of sides or points and the

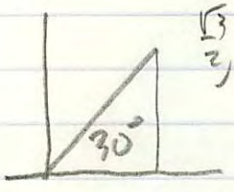
Conjecture
Examination

① $\sec x > 0 \therefore \sec x = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\cos x}$ (C)

② I $f(x) = \sin(x^2-1)$ $f(-x) = f(x)$ even?
II $g(x) = \cos(x^3 - 2)$ $g(-x)$ NO
III $h(x) = (x-3)^2$ $h(-x)$ NO
I only (A)

③ I $f(x) = \sin(x - \frac{\pi}{2})$ NO asym
II $g(x) = \cos x$ YES
III $h(x) = \cot x$ NO at $k\pi$
II only (B)

④



$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (B)

⑤ $y = 3\cos\left(\frac{2}{3}x - \pi\right)$ $\frac{\text{normal period}}{\text{coefficient of } x} = \text{new period}$

$\frac{\pi}{\frac{2}{3}} = \frac{3\pi}{2}$ D

⑥ $f(x) = \frac{\sin x - x}{1 - \cos x}$ $f(-x) = \frac{-\sin x + x}{1 - \cos x}$ C

⑦ $f(x) = \cos x$ odd $f(x) = \sin^2 x$ even $f(x) = \cos|x|$ even $f(x) = \frac{1-\cos x}{\sin x}$

$\frac{1-\cos x}{-\sin x}$

$-f(x)$

odd

$f(x) = \sec x + \csc x$
neither
(E)

(8) $f(x)=x^2$ Dn evaluated at $x=a$: $2a$ (15)

(9) $f(x)=-\frac{3}{x}$ $f'(x)=\frac{3}{x^2}$ $=\frac{1}{3}$ $f(3)=-1$

$x=3$

$(3,-1)$ $m=\frac{1}{3}$

$y+1=\frac{1}{3}(x-3)$

$y=\frac{1}{3}x-2$ (A)

(10) $\lim_{x\to 1} \frac{x-1}{x^2+2x-3}$

$\lim_{x\to 1} \frac{x-1}{(x-1)(x+3)} = \frac{1}{4}$ (E)

(11) $f(x)=2-\left|4+\frac{x}{2}\right|$ $4+\frac{x}{2}=0$

$\frac{x}{2}=-4$

$x=-8$ (D)

(12) $\lim_{x\to -\frac{1}{3}} \frac{9x^2-1}{3x+1}$

$\lim_{x\to -\frac{1}{3}} \frac{(3x+1)(3x-1)}{(3x+1)}$

-2 (D)

(13) $\lim_{x\to 2} \frac{\sqrt{x+14}-4}{x-2}$ $\frac{\sqrt{\ }+4}{\sqrt{\ }+4}$

$\lim_{x\to 2} \frac{x+14-16}{(x-2)(\sqrt{x+14}+4)}$ (C)

$\lim_{x\to 2} \frac{1}{\sqrt{x+14}+4} = \frac{1}{4+4} = \frac{1}{8}$

(14) $f(x) = \dfrac{x-7}{\sqrt{x-3}-2}$

I $\lim_{x\to 7} \dfrac{x-7}{(\sqrt{x-3}-2)} \cdot \dfrac{\sqrt{}+2}{\sqrt{}+2}$

$\lim_{x\to 7} \dfrac{(x-7)(\sqrt{x-3}+2)}{x-3-4} = 4$

True

II $f(7)$ undefined

True. Domain Restriction

III $\lim_{x\to 7} f(x) = f(7)$

ND

$f(7)$ NOT def.

I, II only. (E)

(15) $f(x) = 3x-5 \quad g(x) = 3x^3 - 5x^2 - 3x + 5$

$\lim_{x\to \frac{5}{3}} \dfrac{3x^3-5x^2-3x+5}{3x-5}$

$\lim_{x\to \frac{5}{3}} \dfrac{x^2(3x-5) - (3x-5)}{(3x-5)}$

$\lim_{x\to \frac{5}{3}} x^2 - 1$

$\frac{25}{9} - 1 = \frac{16}{9}$ (B)

(16) $\lim_{x\to 0} \dfrac{3x-1}{2x}$ DNE

$\lim_{x\to 0^+} f(x) \to -\infty$

$\lim_{x\to 0^-} f(x) \to +\infty$.

(E)

(17) $\lim_{x\to 5} \sqrt{\frac{x^2-9}{x-3}}$

$\lim_{x\to 5} \sqrt{x+3} = 2\sqrt{2}$ (B)

(18) $\lim_{x\to 5^-} f(x) = -25+3 = -22$ (C)

(19) $f(x) = \frac{|2x-4|}{x-2}$ $\lim_{x\to 2^+} 2\frac{(x-2)}{x-2} = 2.$ (C)

(20) $\lim_{x\to 2^-} f(x) = 0.$ (D)

(21) I $\lim_{x\to 2^-} f(x)$ DNE No $= 2.$

II $\lim_{x\to 2^+} f(x) = 0$ Yes

III $\lim_{x\to 2} f(x) = 0$ No (C) $\neq$ (B) (E).

(22) $f(x) = \frac{x^2+2x}{x(x+2)^2(x-1)^4}$ $\lim_{x\to a} f(x) = \infty$ if $a \in \{-2, 1\}$

$\frac{x(x+2)}{x(x+2)^2(x-1)^4}$

$\frac{1}{(x+2)(x-1)^4}$

(C)

(23) $f(x) = \dfrac{-2x-5}{2(5-4x^2)} \Rightarrow f(x) = \dfrac{-1}{5+2x}$

$(5-2x)(5+2x)$ Vert asympT at $x = -\frac{5}{2}$ only.

$x = \pm\frac{5}{2}$ is a "Hole." (B)

(24) $\lim_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-25}$

$\lim_{x\to 5^-} \dfrac{(x-5)(x+2)}{(x-5)(x+5)} = \dfrac{7}{10}$ (a)

(25) I $f(x) = \frac{1}{x-2}$ Yes.

II $G(x) = \dfrac{x-2}{x-2}$ NO Hole.

III $H(x) = \begin{cases} \frac{1}{x-2} & \text{if } x < 2 \\ x+5 & \text{if } x \geq 2 \end{cases}$ Yes. (D)

(26) $f(x) = \dfrac{3x^2-2x+5}{7x-2x^2-1}$ Horizontal asympT = like limit as $x \to \infty$ ie divide Denom into Num.

$y = -\frac{3}{2}$ (C)

(27) $\lim_{x\to-\infty} \dfrac{x^2+4}{x^4-1} \to 0$ (a)

(28) $xy-2y = 3x-4$ $(x-2)y = 3x-4$

$y = \dfrac{3x-4}{x-2}$ $\lim_{x\to\infty} y = 3.$

NOTICE HORIZONTAL ASYMPTOTE. $y = 3$ (C)

(29) $\lim_{x\to-\infty}\left(-5x+\frac{3}{2x^3}\right)$

$\lim_{x\to-\infty}\frac{-10x^4+3}{2x^3}$ $\lim_{x\to-\infty}\frac{-10x+\frac{3}{x^3}}{2}\to+\infty$ No limit

(B)

(30) $f(x)=\frac{x^2+2x}{x^2-2x}$ Vert asym at $x=2$ Domain Rest

$=\frac{x(x+2)}{x(x-2)}$ Horizontal asymptote $y=1$ $\lim_{x\to\infty}$

$=\frac{x+2}{x-2}$

(E)

★ (31) $f(x)=\frac{x^2-2}{1-x-x^2}$ $\lim_{x\to\pm\infty} f(x)=-1$.

Very Nice!

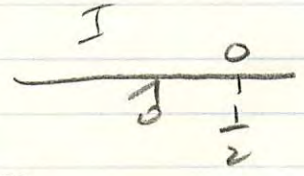
$\frac{x^2-2}{1-x-x^2}\overset{\text{set}}{=}-1$

$x^2-2=x^2+x-1$

$x=-1$

(C)

(D) (32) $f(x)=\frac{x+4}{2x-1}$



$\left|\frac{x+4}{2x-1}-\frac{1}{2}\right|<\frac{1}{10}$

$\left|\frac{2x+8-2x+1}{2(2x-1)}\right|<\frac{1}{10}$

$\left|\frac{9}{2(2x-1)}\right|<\frac{1}{10}$

I $\frac{9}{-2(2x-1)}<\frac{1}{10}$

$90<-4x+2$

$88<-4x$

$-22>x$

II $\frac{9}{2(2x-1)}<\frac{1}{10}$

$90<4x-2$

$92<4x$

$23<x$

of the choices, D is the one, however satisfied if $x<-22$ as well.

Show this as! $\sqrt{x^2(1+\frac{4}{x}+\frac{4}{x^2})}$

(33) $\lim_{x\to\infty} \frac{\sqrt{x^2+4x+4}}{2x+3}$

using absolute value $\sqrt{x^2}\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}$

$\lim_{x\to\infty} \frac{\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}}{2+\frac{3}{x}} \to \frac{1}{2}$ (C)

$\frac{x\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}}{2x+3}$ $\frac{1}{x}$ $\frac{1}{x}$ etc

(34) $f(x) = \begin{cases} 2-3x & \text{if } x<1 \\ -1 & \text{if } x=1 \\ 2x^2-5 & \text{if } x>1 \end{cases}$

I $\lim_{x\to 1^-} f(x) = -1$ II $f(1) = -1$ III $\lim_{x\to 1} f(x) = f(1)$

Yes. Yes. NO $\lim_{x\to 1^+} f(x) = -3$ → no (L) ≠ (R)

I, II only. (D)

(35) $f(x) = \begin{cases} -3ax^2-1 & \text{if } x>1 \\ 3b-2x & \text{if } x\le 1 \end{cases}$

$\lim_{x\to 1^-} f(x) \overset{\text{set}}{=} \lim_{x\to 1^+} f(x)$

$3b-2 = -3a-1$

$3a+3b = 1$

$a+b = \frac{1}{3}$ (a)

(36) $f(x) = \begin{cases} 3kx-5 & \text{if } x>2 \\ 4x-5k & \text{if } x\le 2 \end{cases}$

$6k-5 \overset{\text{set}}{=} 8-5k$

$11k = 13$

$k = \frac{13}{11}$ (B)

$(x+2)(x-1)$

(37) $f(x) = \begin{cases} \frac{x^2+x-2}{x-1} & \text{if } |x| < 1 \\ b & \text{if } x = 1 \end{cases}$

$\lim_{x \to 1}$

$3 = b$ (C)

(38) c may or may not be true

(2) $\lim_{x \to 2} f(x) = 8$ ✓

$\lim_{x \to 2^-} x^2 + 2x = 8$

$\lim_{x \to 2^+} 2x + 4 = 8$

$f(2) = 8$ (1) ✓

(3)

(39)

I Yes polynomial

II NO $x = 1$

III Yes. D_f $x \in \mathbb{R}$, f is cont at $x = 2$ as well.

I, III only. (D)

(40) I $L \neq R$ True

II $\lim_{x \to -2}$ DNE NO

III $f(-2) \neq \lim_{x \to -2^-} f(x)$ True.

(D)

$\lim_{x \to -2^-} -(x^2) - 17$

$-4 - 17$

-21 $L \neq R$

$\lim_{x \to -2^+} -3x^2 - 1$

$-3(4) - 1$

-13.

$f(-2) = -13 \neq 21$

(41) $\lim_{x \to 0} \frac{2}{5} \cdot \frac{\sin 3x}{x} \cdot \frac{3}{3}$

$\lim_{x \to 0} \frac{6}{5} \frac{\sin 3x}{3x}$

$\frac{6}{5} \cdot 1 = \frac{6}{5}$ (B)

(42) $\lim_{x \to \frac{\pi}{4}} \frac{2x - \frac{\pi}{2}}{\cos 2x}$ let $2x - \frac{\pi}{2} = A$

$\therefore 2x = A + \frac{\pi}{2}$

$\lim_{A \to 0} \frac{A}{\cos(A + \frac{\pi}{2})}$

$\lim_{A \to 0} \frac{A}{\cos A \cos\frac{\pi}{2} - \sin A \sin\frac{\pi}{2}}$

$\lim_{A \to 0} \frac{A}{-\sin A} = -1$ D

A (43) $\lim_{x \to \infty} \frac{\sin x}{2x} = 0$

$\lim_{x \to \infty} \frac{-2}{2x} \leq \lim_{x \to \infty} \frac{\sin x}{2x} \leq \lim_{x \to \infty} \frac{2}{2x}$

$0 \leq \lim_{x \to \infty} \frac{\sin x}{2x} \leq 0$

zero.

(44) $\lim_{x \to 0} \frac{\cos 2x}{x}$

$\lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \frac{2}{\cos 2x}$

$1 \cdot \frac{2}{1} = 2$ (E)

(45) $\lim_{x \to \frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x)}{\frac{\pi}{2} - x}$ let $\frac{\pi}{2} - x = A$

$\lim_{A \to 0} \frac{\sin A}{A} = 1$ (E)

(46) $\lim_{x \to 0} \sin\frac{1}{x} \cos\frac{1}{x}$ (E) DNE

(47) I $f(x) = 2x^{\frac{3}{2}}$ II $g(x) = \frac{3}{2x}$ III $h(x) = 5x^{\frac{2}{5}} - 3$.

Yes, NO $x \neq 0$ NO D_x DNE at $x=0$

I only. (A)

(48) Same as 47 if Differ everywhere This is also Cont everywhere.

(A)

Very Good pblm (49) $f(x) = \begin{cases} 3x^2 & x \leq -1 \\ ax+b & x \geq -1 \end{cases}$

1st f must be cont $\therefore \lim_{x\to -1^-} 3x^2 \overset{\text{must}}{=} \lim_{x\to -1^+} ax+b$

$3 = -a+b$

2nd $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$\lim_{x\to -1^-} \frac{3x^2-(-a+b)}{x+1}$ $\lim_{x\to -1^+} \frac{ax+b-(-a+b)}{x+1}$

$\lim_{x\to -1^-} \frac{3x^2-3}{x+1}$ $\lim_{x\to -1^+} \frac{ax+b+a-b}{x+1}$

$\lim_{x\to -1^-} 3(x-1)$ $\lim_{x\to -1^+} \frac{a(x+1)}{(x+1)} = a$

-6

$\therefore a = -6$

$3 = -(-6)+b$

$3 = 6+b$

$b = -3$

$a+b = -6-3$

$= -9$

(B).

(50) $f(x) = \begin{cases} 3x^2+4 & x<2 \\ bx^3 & x\geq 2 \end{cases}$

$\lim_{x\to 2^-} 3x^2+4 = \lim_{x\to 2} bx^3$

$16 \overset{\text{Set}}{=} 8b$

$b=2$ (E)

Good ph/m

(51) $f(x) = \begin{cases} 3x^2+4x & x\leq 1 \\ 2x^3+bx+c\,; & x>1 \end{cases}$

Cont: $\lim_{x\to 1^-} 3x^2+4x \overset{\text{Set}}{=} \lim_{x\to 1^+} 2x^3+bx+c$

$7 = 2+b+c$

$b+c=5$

Der. $\lim_{x\to 1^-} \frac{3x^2+4x-(3+4)}{x-1}$

$\lim_{x\to 1^-} \frac{3x^2+4x-7}{x-1}$

$\lim_{x\to 1^-} \frac{(3x+7)(x-1)}{(x-1)}$

10

$\lim_{x\to 1^+} \frac{2x^3+bx+c-7}{x-1}$

$\lim_{x\to 1^+} \frac{2x^3+bx+c-(2+b+c)}{x-1}$

$\lim_{x\to 1^+} \frac{2x^3+bx+c-2-b-c}{x-1}$

$\lim_{x\to 1^+} \frac{2x^3+bx-2-b}{x-1}$

$\lim_{x\to 1^+} \frac{2(x^3-1)+b(x-1)}{x-1}$

$\lim_{x\to 1^+} \frac{2(x^2+x+1)+b}{} =$

$6+b \overset{\text{Set}}{=} 10$

$b=4.$

(C)

(52) $f(x) = |x^3|$

NO I f is sym WRT origin NO f is even not odd

II cont? Yes II & IV.

III f diff? ← needs to be proven

using Def of Der:

$$\lim_{x\to 0^-} \frac{-x^3}{x} \qquad \lim_{x\to 0^+} \frac{x^3}{x}$$

$$\lim_{x\to 0^-} -x^2 = 0 \qquad \lim_{x\to 0^+} x^2 = 0$$

Der Does exist at $x=0$.

(53) Domain of min f(x).

$x \in [-2, 5)$ (1)